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# Sensitivity analysis of the transient energy function method

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# Sensitivity analysis
# of the transient energy function method

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# 1 INTRODUCTION

## 1.1 Power System Security

The main concern of electric utilities is to supply the customers with inexpensive electricity at admissible voltage and frequency without interruption of service. To achieve this goal, it is vital to maintain the security of the bulk power system.

The security of power system is the state of the system which ensures the integrity of power system for a set of disturbances such as faults and outages of power system component. At present large portions of the North-American interconnected power system are being operated at power transfer levels in excess of anticipated design goals in order to obtain economic benefits from firm energy transactions. There are indications that this trend will continue due to limited availability of new transmission right-of-ways. This mode of operation requires more elaborate and sophisticated operations planning, due to the unavailability of ample security margin.

Power system security is important to planners as well as to operators. For planners, security is an aspect of reliability, because the planner is responsible for designing a system that will be reliable over a long period of time.

On the other hand, the operator is responsible for maintaining the the actual functioning of the real system that is changing as well as is being subject to changing demands and changing disturbances from the environment. So security is an instantaneous, time dependent measure of the robustness of the system with respect to imminent disturbances and available reserve actions [1,2].

## 1.2 Tools for Security Assessment

Usually security assessment is categorized into either static security assessment or dynamic security assessment. For a stability limited power system, dynamic security assessment ensures that the system remains in synchronism in the transition to an acceptable operating state for a predetermined set of disturbances. Static security ensures that the line flows do not exceed the thermal rating for their lines and bus voltages remain within acceptable ranges.

Though the predetermined line loading limits and stability loading limits from off-line studies can guide the operator in general situations, these are not sufficient for security assessment under ever-changing system operating condition and environment when the system is stability limited. As a consequence several off-line tools have been modified and used as tools for on-line situations. Optimal power flow and static security assessment program are examples.

Traditionally the time domain simulation using digital computers is accepted as a practical method of large disturbance stability analysis for off-line dynamic security assessment. This method is robust and reliable but it is computationally intensive and time consuming. Also the transient stability programs currently in use do not fully exploit the information available from the results of time-domain simulations due to extensive demand for skilled manpower for output analysis, as a result they are not suited for on-line analysis.

It is very difficult to obtain sensitivity information from the output of a conventional time domain study with regard to critical system parameters and quantities, since the time domain solution is conducted based on a specific scenario. If the network parameters or operating conditions are changed, the analysis has to be repeated.

In a typical operations environment, the system operator needs to define stability limits and evaluate margins to signal the approach of vulnerable situations under ever-changing conditions. In addition there is also need to recognize approaching vulnerability and take appropriate preventive action.

Modern energy control centers presently include application software for static security assessment with respect to contingencies involving steady state operation following a disturbance. This assumes that the system has survived the transition during the contingency. This assumption was made in the past mainly due to lack of tools to assess transient stability in an on-line situation.

In contrast to the time domain approach the direct methods determine the stability of the power system without explicitly solving the differential equations describing the dynamics of the system. Moreover, they provide a qualitative measure of the degree of stability which can be analyzed as a function of important system parameters such as generation shifts among generators, power flows in critical lines and changes in the load.

Because of the above mentioned merits, direct methods are appealing and have been the subject of research efforts since the early work of A.A.Gorev [3] in the 1930's.

## 1.3 Direct Methods

### 1.3.1 Origin of the direct methods

All direct methods of stability assessment are directly or indirectly related to Lyapunov's direct method and Hamiltonian Mechanics. The main idea of Lyapunov's direct method is that " if the system dynamics are such that the energy of the system is non-increasing with time and does not exceed a certain threshold value" this gives sufficient condition for the stability of the equilibrium point. The theorem of Lyapunov put this idea into mathematically precise terms.

The well known equal area criterion [4] which is a direct method for a two machine system, explains the energy conversion process during transients. For a two machine system, it is very easy to formulate a Lyapunov function which is the sum of potential energy and kinetic energy and is the Hamiltonian.

But difficulty arises when we apply direct methods to multi-machine power systems. During the early stages of research the following reasons have prevented energy type direct methods from being applied to multi-machine power system stability assessment.

Firstly, kinetic energy of generator rotors is dependent on the coordinates chosen. Proper choice of the coordinate is vital in identifying the kinetic energy which is converted into potential energy when the disturbed machines are climbing up the potential barrier during the system separation process.

Secondly, the potential energy profile, created by the machine angle deviation from the post-disturbance stable equilibrium point, is disturbance dependent. In other words the potential energy profile is a function of the trajectories of the machine angles which we do not want to compute in direct method to assess stability. Thus, in order to compute the potential energy, we have to identify the trajectories of the critical machines without solving dynamic equations.

Thirdly, when the number of generators increases, the numerical burden increases rapidly.

In the 1930's Gorev [3] used the first integral of energy to obtain a criterion for stability. But there was no further accomplishment after that. The first major work on the subject in English was by Magnusson in 1947 [5]. In these early works, the power system was considered as a conservative system where the total energy in the system is conserved.

In 1958, Aylett proposed an energy-integral criterion to obtain the transient stability limit [6]. The most significant aspect of Aylett's work is the formulation of the system equations based on the inter-machine movements. This is in accord with the the physical dynamic behavior of the machines which determines whether synchronism is maintained. But it was too difficult to apply this method to multi-machine power systems, because of the enormous numerical burden involved in determining the particular singular point from the $(2^{n-1}-1)$ singular points, which theoretically exist for a n-machine system.

### 1.3.2 Progress and improvement of direct method

In 1966, El-Abiad and Nagappan [7] proposed a procedure of assessing the transient stability region of a multi-machine power system. In their approach the transfer conductances of the power system were included in formulating the Lyapunov function. They manipulated the energy terms corresponding to transfer conductances to be integrable analytically. Incorporation of transfer conductances requires computing path-dependent integrals which can not be computed without knowing the trajectories.

It is interesting to note that the authors argue that their integral function is a Lyapunov function even though it is not (It is indefinite in the neighborhood of the stable equilibrium point.). But this mathematical contradiction does not pose a severe problem, because we assume that the point under consideration is the stable equilibrium point and our main concern is to obtain the largest region of attraction. The procedure in this work to assess the stability is still used in the transient energy function method.

Many researchers in the 1970's have neglected transfer conductances on the basis that these are small. But this assumption was not good, since the constant impedance loads are reflected in the transfer conductance terms of the network matrix. Uemura et al. [8] suggested a linear trajectory approximation, which is currently used in practice. The authors concluded that if a given multi-machine system swings like a two-machine system, then the energy function obtained by the linear trajectory approximation method will yield an approximately good result.

Tavora and Smith [9] developed the concept of the center of inertia (COI). This work vastly improved the formulation of the stability problem, and resulted in neglecting the portion of energy involved in accelerating the COI, contributing to stability. These properties were also identified in [10].

In 1976, Gupta and El-Abiad [11] made an important contribution in identifying the fault-trajectory dependent unstable equilibrium point (UEP). Until then the critical energy was calculated at the UEP having the lowest energy level.

In 1979, Athay and co-workers at System Control, Inc. (SCI) [12] made significant progress toward the development of transient energy function (TEF) method for application in transient stability analysis. These accomplishments are summarized below.

1. Their works showed that the relevant UEP was determined mainly by fault-trajectory. The most weakly connected generators may not lose synchronism.
2. The concept of Potential Energy Boundary Surface(PEBS), developed by Kakimoto and co-workers [13] , was utilized to understand system separation mechanism and to determine the fault-trajectory dependent UEP.
3. The development of a formalism for the Transient Energy Stability Analysis (TESA) approach which was based of a Lyapunov theory that involves the concepts of invariant sets.

In the early 1980s Fouad and coworkers at Iowa State University [14,15,16] made several contributions to this area of research. The following is a summary of the main accomplishments of their work [1].

1. The concept of controlling (or relevant) UEP is valid.
2. The critical trajectory of the critical generators is controlled by the controlling UEP.
3. Instability is determined by the gross motion of the critical generators.

[1]This technique is called TEF method in the literature ever since

4. A significant portion of the kinetic energy which does not contribute to the system separation is identified. By correcting the kinetic energy the stability assessment becomes less conservative.

5. For practical purposes, the critical energy is equal to the energy level at the controlling UEP.

6. The energy margin is an indicator of the robustness of the power system. It allows the ranking of contingent disturbances for a given operating condition.

Further investigation followed in order to identify the complex mode of disturbance, because observance of the fault trajectory shows that not all the severely disturbed machines lose synchronism. Fouad et al. [17] developed a reliable and fast technique to determine the controlling UEP by identifying the weakest link. This procedure significantly improved the prospect of the transient energy function method.

As a result of these various advances, the conservativeness of the TEF method has been significantly reduced.

### 1.3.3 Application of the transient energy function method

The next phase of the research efforts consists of two aspects:

- New application of the TEF method
- Incorporation of new models in the TEF method.

Associated with both there were other issues, e.g., computational problems. The achievements of these research works are:

1. Determination of generation shedding requirement using the TEF method [18].
2. Dynamic security assessment by determining critical interface power flow limits [19].
3. The application of the TEF method to large scale-scale power systems [20].
4. Incorporating out-of-step impedance relay in the TEF method [21].
5. Incorporating the effect of the exciter in the TEF method [22].
6. Incorporating the two-terminal HVDC lines in the TEF method [23].
7. Incorporating the non-linear load in the TEF method[20].

Current research efforts are concentrated on the sensitivity analysis of the TEF method and the application of the TEF method to stressed power system.

## 1.4 Statement of the Problem

### 1.4.1 Motivation of the study

In system operation, static security assessment is usually carried out by on-line simulation of critical contingencies to ensure that bus voltage limits and thermal limits will not be exceeded. On the other hand, overall dynamic security assessment is not presently computed on-line. Fast and very reliable techniques are essential for on-line dynamic security assessment in order to determine, in real time, the stable regimes and conditions under ever changing system dispatch as well as scheduled or forced equipment outages in the system. In stability limited networks, several hundred contingencies have to be assessed within seconds for variation in dispatch and power flows, to determine secure regimes of operation.

Given these safe limits the system operator would take the necessary actions in order to remain within the normal state and thus act to prevent stability crises. For example, if a storm approaches a certain area , the system operator can shift generation between critical and non-critical machine groups in order to remain within the boundary of the safe operating region.

Due to continued development and enhancement, the TEF method now provides accurate and reliable stability assessment. For the TEF method to be an effective tool for dynamic security assessment, an important step is to relate the energy margin and relevant system variables, such as generation change, load change and interface power flow change. With these sensitivity information a system operator can fully exploit the result of stability assessment and develop a constructive knowledge for operating his system safely.

### 1.4.2 Scope of the work

In this research work a procedure to determine transient stability limits for particular contingency using analytic sensitivity of the energy margin is developed. The objectives of this research work are as follows.

1. Development of a dynamic sensitivity model to obtain sensitivity of the energy margin with respect to generation shifts.

2. Apply the proposed technique to determine critical generator loadings and critical transmission interface power flow limits.

3. Conduct simulation studies to test and validate the procedure developed on realistic power systems.

# 2 THE TRANSIENT ENERGY FUNCTION METHOD FORMULATION

## 2.1 The Transient Energy Function Formulation

### 2.1.1 The classical power system model

The classical model of power system usually used in transient stability studies is reasonably accurate for first swing stability analysis. This model is the simplest model used in power system dynamics, and requires a minimum amount of data. The following assumptions are made for classical model of power systems [24].

1. Mechanical power input to each generator is held constant.
2. Damping or asynchronous power is negligible.
3. The synchronous machines are modeled as constant voltage sources behind the transient reactance.
4. The motion of machine rotor angle coincides with the angle of the voltage behind the transient reactance.
5. Loads can be represented by passive impedances.

In addition to these five assumptions electrical transients in transmission network including machine stator circuit are usually neglected assuming its time constant is very small. Using the above model, the dynamic equations of an n-machine system during the transient period are given by

$$\begin{aligned} M_i\dot{\omega}_i &= P_i - P_{ei} \\ \dot{\delta}_i &= \omega_i\ , \qquad\qquad i = 1,2,3,...,n, \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} P_{ei} &= \sum_{\substack{j=1 \\ j\neq i}}^{n} [C_{ij}\sin(\delta_i - \delta_j) + D_{ij}\cos(\delta_i - \delta_j)] \\ P_i &= P_{mi} - E_i^2 G_{ii} \end{aligned} \tag{2.2}$$

and

$C_{ij} \;=\; E_i E_j B_{ij}$,

$D_{ij} \;=\; E_i E_j G_{ij}$,

$P_{mi}$ ; the mechanical power input to generator $i$,

$E_i$ ; the internal voltage of machine $i$,

$M_i$ ; the inertia constant of machine $i$,

$\delta_i$ ; the electrical angle of machine $i$ with respect to a synchronously rotating reference frame,

$\omega_i$ ; the electrical angular speed of machine $i$ with respect to a synchronously rotating reference frame,

$G_{ij}, B_{ij}$ ; the real and imaginary components of the $ij-th$ element of the reduced admittance matrix.

The network admittance matrix used here has been reduced to the internal generator nodes. Also all variables are in per unit.

### 2.1.2 The center of inertia reference frame

Major breakthrough in the TEF development, was achieved using the center of inertia(COI) transformation [9,10]. The position of the center of inertia is defined by

$$COI \equiv \delta_0 \equiv \frac{1}{M_T} \sum_{i=1}^{n} M_i \delta_i, \tag{2.3}$$

where

$$M_T = \sum_{i=1}^{n} M_i.$$

Then the motion of the center of inertia is determined by

$$M_T \dot{\omega}_0 = P_{COI} \tag{2.4}$$

where

$$\begin{aligned} P_{COI} &= \sum_{i=1}^{n} (P_i - P_{ei}), \\ \dot{\delta}_0 &= \omega_0 \end{aligned}$$

By adopting the position of center of inertia as the reference frame, we have

$$\begin{aligned} \theta_i &= \delta_i - \delta_0, \\ \tilde{\omega}_i &= \dot{\theta}_i = \omega_i - \omega_0, \end{aligned} \tag{2.5}$$

then the equations of motion of the generators in the COI reference frame become

$$\begin{aligned} M_i\dot{\tilde{\omega}}_i &= P_i - P_{ei} - \frac{M_i}{M_T}P_{COI}, \\ \dot{\theta}_i &= \tilde{\omega}_i\ , \qquad\qquad i = 1,2,3,...,n. \end{aligned} \tag{2.6}$$

By definition the COI coordinates, $\theta_i$ and $\tilde{\omega}_i$ are not linearly independent and satisfy the following important relationship.

$$\begin{aligned} \sum_{i=1}^{n} M_i\theta_i &= 0, \\ \sum_{i=1}^{n} M_i\tilde{\omega}_i &= 0. \end{aligned} \tag{2.7}$$

By adopting COI coordinates, only the kinetic energy related to the asynchronous motion of the rotors from the collective motion of fictitious inertia center, is identified as kinetic energy responsible for system separation.

The equilibrium points of the dynamic equation(2.6) are the points which satisfy the following condition.

$$\begin{aligned} P_i - P_{ei} - \frac{M_i}{M_T}P_{COI} &= 0 \\ \tilde{\omega}_i &= 0\ , \qquad\qquad i = 1,2,...,n \end{aligned} \tag{2.8}$$

Notice that from equation(2.8) equilibrium can be reached with $P_{COI}$ having non-zero value. This means that position of the center of inertia can accelerate while the generators remain in synchronism.

### 2.1.3 Transient energy function

From equation(2.6), system transient energy of the post-disturbance network can be obtained. The post-disturbance swing equation of machine i is multiplied by $\dot{\theta}_i$ and summed up for the n machines in the system:

$$\sum_{i=1}^{n} [M_i\dot{\tilde{\omega}}_i - P_i + P_{ei} + \frac{M_i}{M_T}P_{COI}]\dot{\theta}_i \tag{2.9}$$

The above expression is integrated with respect to time, using as a lower limit $t = t_s$ , where $\underline{\tilde{\omega}}(t_s) = 0$ and $\underline{\theta}(t_s) = \underline{\theta}^s$ , then the transient energy function V takes the following form.

$$\begin{aligned} V &= \frac{1}{2}\sum_{i=1}^{n} M_i\tilde{\omega}_i^2 - \sum_{i=1}^{n} P_i(\theta_i - \theta_i^s) - \\ &\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} [C_{ij}(\cos\theta_{ij} - \cos\theta_{ij}^s) - \int_{\theta_i^s+\theta_j^s}^{\theta_i+\theta_j} D_{ij}\cos\theta_{ij}d(\theta_i + \theta_j)] \end{aligned} \tag{2.10}$$

where

$$\begin{aligned} \underline{\theta}^s &= \text{stable equilibrium point of post-disturbance network,} \\ \theta_{ij} &= \theta_i - \theta_j \end{aligned}$$

Physically the first term of equation(2.9) is the sum of rotor kinetic energy of the generators. The other terms constitute the potential energy.

Since the last term in equation(2.10) consists of a path dependent integral which can be evaluated if the system trajectory is known, approximation of the system trajectory is required to calculate this term. By using a linear trajectory approximation [8] equation(2.9) becomes

$$
\begin{aligned}
V = {} & \frac{1}{2}\sum_{i=1}^{n} M_i\tilde{\omega}_i^2 - \sum_{i=1}^{n} P_i(\theta_i - \theta_i^s) - \\
& \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} [C_{ij}(\cos\theta_{ij} - \cos\theta_{ij}^s) - I_{ij}] \qquad (2.11)
\end{aligned}
$$

where

$$
\begin{aligned}
I_{ij} &= D_{ij}\frac{\theta_i + \theta_j - \theta_i^s - \theta_j^s}{\theta_{ij} - \theta_{ij}^s}(\sin\theta_{ij} - \sin\theta_{ij}^s) \\
&= \text{an approximation of the transfer conductance terms } (\mathrm{G_{ij}}) \qquad (2.12)
\end{aligned}
$$

## 2.2 Stability Assessment by the TEF Method

### 2.2.1 Critical energy and the energy margin

The transient energy can be evaluated between any two points along the system trajectory. For example, for a faulted system at the instant of clearing, $(\underline{\theta}, \underline{\tilde{\omega}}) = (\underline{\theta}^{cl}, \underline{\tilde{\omega}}^{cl})$, the transient energy with respect to post-disturbance stable equilibrium point $\underline{\theta}^s$ is given by

$$
\begin{aligned}
V_{cl} &= V|_{\underline{\theta}^s}^{\underline{\theta}^{cl}} \\
&= \frac{1}{2}\sum_{i=1}^{n} M_i(\tilde{\omega}_i^{cl})^2 - \sum_{i=1}^{n} P_i(\theta_i^{cl} - \theta_i^s)
\end{aligned}
$$

$$-\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}[C_{ij}(\cos\theta_{ij}^{cl}-\cos\theta_{ij}^{s})-I_{ij}^{cl}] \tag{2.13}$$

where

$$I_{ij}^{cl} = D_{ij}\frac{\theta_i^{cl}+\theta_j^{cl}-\theta_i^{s}-\theta_j^{s}}{\theta_{ij}^{cl}-\theta_{ij}^{s}}(\sin\theta_{ij}^{cl}-\sin\theta_{ij}^{s}) \tag{2.14}$$

The critical transient energy is defined as transient energy level at controlling unstable equilibrium point $\underline{\theta}^u$ with $\underline{\tilde{\omega}} = 0$.

$$\begin{aligned} V_{cr} &= V_u = V|_{\underline{\theta}^s}^{\underline{\theta}^u} \\ &= -\sum_{i=1}^{n} P_i(\theta_i^u-\theta_i^s) - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}[C_{ij}(\cos\theta_{ij}^{u}-\cos\theta_{ij}^{s})-I_{ij}^{u}] \end{aligned} \tag{2.15}$$

where

$$I_{ij}^{u} = D_{ij}\frac{\theta_i^{u}+\theta_j^{u}-\theta_i^{s}-\theta_j^{s}}{\theta_{ij}^{u}-\theta_{ij}^{s}}(\sin\theta_{ij}^{u}-\sin\theta_{ij}^{s}) \tag{2.16}$$

The system transient energy margin $\Delta V$is obtained from the difference of these two values.

$$\Delta V = V_{cr} - V_{cl} \tag{2.17}$$

Substituting for $V_{cl}$ and $V_{cr}$ in equation (2.13), we can show that transient energy margin can be approximated as

$$\begin{aligned}\Delta V &= V|_{\underline{\theta}^{cl}}^{\underline{\theta}^{u}} \\ &= -\frac{1}{2}\sum_{i=1}^{n} M_i(\tilde{\omega}_i^{cl})^2 - \sum_{i=1}^{n} P_i(\theta_i^u - \theta_i^{cl}) \\ &\quad -\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}[C_{ij}(\cos\theta_{ij}^u - \cos\theta_{ij}^{cl}) - I_{ij}|_{\underline{\theta}^{cl}}^{\underline{\theta}^{u}}]\end{aligned} \tag{2.18}$$

where

$$I_{ij}|_{\underline{\theta}^{cl}}^{\underline{\theta}^{u}} = D_{ij}\frac{\theta_i^u + \theta_j^u - \theta_i^{cl} - \theta_j^{cl}}{\theta_{ij}^u - \theta_{ij}^{cl}}(\sin\theta_{ij}^u - \sin\theta_{ij}^{cl}) \tag{2.19}$$

Notice that equation (2.18) involves less interval of approximation than equation(2.13), (2.15) and (2.17). In equation (2.18) the trajectory is approximated from the fault clearing point to the controlling UEP.

### 2.2.2 The kinetic energy correction

After a major disturbance, a power system tends to split into two groups of machines. In such a condition not all kinetic energy at the instant of fault clearing contributes to the separation of the critical machines from the the rest of the system. The transient kinetic energy which is responsible for the separation of the critical group from the rest of the system is identified as the kinetic energy associated with the gross motion of the critical group with respect to inertial center of the rest of the system. The remaining portion of the kinetic energy is not converted to the other form of energy for stability to be maintained. It is identified as the inter-machine motion kinetic energy in each of the group. To account for this fact, the kinetic energy is corrected as follows [14,16]

$$V_{KE}^{corr} = \frac{1}{2} M_{eq} \tilde{\omega}_{eq}^2 \tag{2.20}$$

where

$$\begin{aligned} M_{eq} &= \frac{M_{cr} M_{sys}}{M_{cr} + M_{sys}}, \\ \tilde{\omega}_{eq} &= \tilde{\omega}_{cr} - \tilde{\omega}_{sys}, \end{aligned}$$

and

$$\begin{aligned} M_{cr} &= \sum_{i \in cr} M_i, \\ M_{sys} &= \sum_{i \in sys} M_i, \end{aligned}$$

$cr$ ; index set of critical generators,

$sys$ ; index set of non-critical generators

$$\begin{aligned} \tilde{\omega}_{cr} &= \Big(\sum_{i \in cr} M_i \tilde{\omega}_i\Big)/M_{cr}, \\ \tilde{\omega}_{sys} &= \Big(\sum_{i \in sys} M_i \tilde{\omega}_i\Big)/M_{sys}. \end{aligned}$$

Then the original kinetic energy term should be replaced with $V_{KE}^{corr}$. Therefore, the energy margin becomes

$$\Delta V = -\frac{1}{2}M_{eq}\tilde{\omega}_{eq}^2 - \sum_{i=1}^{n} P_i(\theta_i^u - \theta_i^{cl}) - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}[C_{ij}(\cos\theta_{ij}^u - \cos\theta_{ij}^{cl}) - I_{ij}|_{\underline{\theta}^{cl}}^{\underline{\theta}^u}] \quad (2.21)$$

### 2.2.3 Stability assessment by the TEF method

In the TEF method the energy margin plays a major role in assessing the power system stability for a particular disturbance. To accurately compute the energy margin, identifying the correct controlling UEP is essential. Theoretically the controlling UEP is the point which the critically cleared system trajectory reaches with zero speed for a particular disturbance. The determination of the "Mode of Disturbance" identifies this pont [17].

Based on the sign of the energy margin, the stability of the power system is easily assessed. If the energy margin is positive, the system is stable; otherwise it is unstable. In the following chapters, the change of energy margin is related to generation changes or transmission interface power flow changes assuming that the total generation is held constant.

# 3 SENSITIVITY OF THE ENERGY MARGIN WITH RESPECT TO GENERATION SHIFTS

## 3.1 Introduction

An inherent advantage of the TEF method is the availability of the energy margin which can be analyzed as a function of relevant system variables such as, generation shifts among generators and power flow in key tie-lines.

From the early stage of the TEF method development this aspects was recognized [16]. In that approach additional disturbance as a form of generation increase was introduced to consume the transient energy margin to assess the relative severity of the impact of additional disturbance. Various research efforts followed after this pioneering work.

Sauer et al. [25] used numerical sensitivity of the energy margin with respect to total system load to derive a stability-limited load supply capability which was incorporated as a constraint in the optimal power flow problem.

El-Kady et al. [26] developed a transient energy margin sensitivity technique combined with power flow distribution factors for fast computation of transmission interface power flow limits by using numeric sensitivity coefficients.

In [27], Vittal et al. assuming linear behavior of the energy margin with respect to generation shifts, developed a technique to determine critical plant loading limits when increased loading is desired for economy, or decreased loading is desired to maintain stability.

Recently Pai et al. [28] proposed an analytic method for determining a maximum load capability of simultaneous interchange capability of the system assuming a certain contingency. Several simplifying assumptions were made ;

- Transfer conductances were neglected.
- Only self clearing faults with no line switching were assumed.

This method successfully related energy margin and maximum load supply capacity. This technique used dynamic sensitivity equations to obtain sensitivity information about the clearing angles and the clearing speeds.

In power system operation under stability-limited condition, the preventive action, generally consists of generation shifts among generators, load shedding or generation rejection. In this chapter sensitivity of the energy margin with respect to generation shifts is obtained by developing the dynamic sensitivity equations [29,30]. Generation shifts result in a new power flow solution,i.e., new equilibrium points. The difference of these two power flow solutions serves as an initial condition variation in the dynamic system [30].

In the analytic sensitivity study, infinitesimal variation is assumed. Therefore direct application of the estimate for infinitesimal variation to the case of small variations, makes it possible to obtain approximate results with a certain degree of accuracy. The issue of accuracy will be treated in a later chapter.

### 3.2 Theory and Description of the Sensitivity Analysis

In the TEF method the energy margin $\Delta V$ is given by equation(2.9), and is rewritten here for convenience.

$$\Delta V = -\frac{1}{2}M_{eq}(\tilde{\omega}_{eq}^{cl})^2 - \sum_{i=1}^{n} P_i(\theta_i^u - \theta^{cl}) - \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} C_{ij}(\cos\theta_{ij}^u - \cos\theta_{ij}^{cl})$$
$$+ \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \frac{\theta_i^u + \theta_j^u - \theta_i^{cl} - \theta_j^{cl}}{\theta_{ij}^u - \theta_{ij}^{cl}} D_{ij}(\sin\theta_{ij}^u - \sin\theta_{ij}^{cl}) \qquad (3.1)$$

where

$P_i = P_{mi} - |E_i|^2 G_{ii}$

$C_{ij} = |E_i||E_j|B_{ij}$

$D_{ij} = |E_i||E_j|G_{ij}$

$P_i$ ; the net mechanical input power at i-th machine terminal

$P_{mi}$ ; the mechanical input power at i-th machine terminal

$G_{ij}$ ; the real part of ij element of internal node reduced post-disturbance admittance matrix

$B_{ij}$ ; the imaginary part of ij element of internal node reduced post-fault admittance matrix.

$E_i$ ; the internal constant voltage source behind transient reactance of machine i

$M_i$ ; the inertia constant of machine i

$\theta_i^{cl}$ ; the clearing angle of the i-th machine rotor in COI reference frame

$\theta_i^u$ ; the controlling UEP angle of the i-th machine rotor in COI reference frame

$\theta_{ij}^{cl} = \theta_i^{cl} - \theta_j^{cl}$

$\theta_{ij}^u = \theta_i^u - \theta_j^{cl}$

$$\tilde{\omega}_{eq}^{cl} = \tilde{\omega}_{cr}^{cl} - \tilde{\omega}_{sys}^{cl}$$

$$\tilde{\omega}_{cr}^{cl} = \frac{1}{M_{cr}} \sum_{i \in cr} M_i \tilde{\omega}_i^{cl}$$

$$M_{cr} = \sum_{i \in cr} Mi$$

$$\tilde{\omega}_{sys}^{cl} = -\frac{1}{(M_T - M_{cr})} \sum_{i \in cr} M_i \tilde{\omega}_i^{cl}$$

$$M_T = \sum_{i=1}^{n} M_i$$

$\tilde{\omega}_i^{cl}$ ; the clearing speed of the i-th machine rotor in COI reference frame

$cr$ ; index set of critical generators

$sys$ ; index set of non-critical generators

In general $\Delta V = \Delta V(\underline{P}_m, \underline{\theta}^u, \underline{\theta}^{cl}, \dot{\underline{\theta}}^{cl}, \underline{E}, B_{ij}, G_{ij})$. When the generation is shifted among generators, the clearing speeds, clearing angles, unstable equilibrium point and the constant voltages behind transient reactance $E_i$ (which is assumed constant during the transient) will change. In reality the reduced admittance matrix terms $B_{ij}$ and $G_{ij}$ will also change as the load bus voltages change in the pre-disturbance power flow solution, and the admittance corresponding to the load changes. However, it is assumed that these changes are small and neglected. These changes will cause the energy margin to vary. If we assume the following ;

1. Total generation is constant

$$\sum_{k=1}^{N-1} \Delta P_{mk} + \Delta P_{mN} = 0,$$

where

$\Delta P_{mk}$ ; variation of mechanical power input at k-th machine

$\Delta P_{mN}$ ; variation of mechanical power input at reference machine(usually has the largest inertia).

$N$ ; the number of generators at which generation is adjusted.

2. Generation shifts do not alter the mode of disturbance.

3. Generation shifts $\Delta P_{mk}$ is not large.

The variation of energy margin $\Delta(\Delta V)$ caused from generation changes can then be approximated as [27,31]

$$\Delta(\Delta V) \approx \sum_{k=1}^{N} \frac{\partial(\Delta V)}{\partial P_{mk}} \Delta P_{mk} \tag{3.2}$$

The sensitivity of the energy margin to change in generation change at the k-th machine is given by the partial derivative of $\Delta V$ with respect to $P_{mk}$. Differentiating equation (3.1) using the chain rule of differentiation, we get

$$\begin{aligned}
\frac{\partial(\Delta V)}{\partial P_{mk}} = & -M_{eq}\tilde{\omega}_{eq}^{cl}\dot{u}_{eq,k}^{cl} - (\theta_k^u - \theta_k^{cl}) - \sum_{i=1}^{n}(P_{mi} - G_{ii}|E_i|^2)(u_{ik}^u - u_{ik}^{cl}) \\
& + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} C_{ij}[\sin\theta_{ij}^u(u_{ik}^u - u_{jk}^u) - \sin\theta_{ij}^{cl}(u_{ik}^{cl} - u_{jk}^{cl})] \\
& + \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} D_{ij}(\sin\theta_{ij}^u - \sin\theta_{ij}^{cl})[\frac{(u_{ik}^u + u_{jk}^u - u_{ik}^{cl} - u_{jk}^{cl})}{(\theta_{ij}^u - \theta_{ij}^{cl})} \\
& - \frac{(u_{ik}^u - u_{jk}^u - u_{ik}^{cl} + u_{jk}^{cl})(\theta_i^u + \theta_j^u - \theta_i^{cl} - \theta_j^{cl})}{(\theta_{ij}^u - \theta_{ij}^{cl})^2}]
\end{aligned}$$

$$+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{(\theta_i^u+\theta_j^u-\theta_i^{cl}-\theta_j^{cl})}{(\theta_{ij}^u-\theta_{ij}^{cl})}D_{ij}[\cos\theta_{ij}^u(u_{ik}^u-u_{jk}^u)$$
$$-\cos\theta_{ij}^{cl}(u_{ik}^{cl}-u_{jk}^{cl})]$$
$$+2\sum_{i=1}^{n}|E_i|\frac{\partial|E_i|}{\partial P_{mk}}G_{ii}(\theta_i^u-\theta_i^{cl})$$
$$-\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(\frac{\partial|E_i|}{\partial P_{mk}}|E_j|+|E_i|\frac{\partial|E_j|}{P_{mk}})B_{ij}(\cos\theta_{ij}^u-\cos\theta_{ij}^{cl})$$
$$+\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}[G_{ij}(\frac{\partial|E_i|}{\partial P_{mk}}|E_j|+|E_i|\frac{\partial|E_i|}{\partial P_{mk}})\frac{(\theta_i^u+\theta_j^u-\theta_i^{cl}-\theta_j^{cl})}{(\theta_{ij}^u-\theta_{ij}^{cl})}$$
$$(\sin\theta_{ij}^u-\sin\theta_{ij}^{cl})] \qquad (3.3)$$

where

$$\dot{u}_{eq,k}^{cl} = \dot{u}_{cr,k}^{cl}-\dot{u}_{sys,k}^{cl}$$
$$\dot{u}_{cr,k}^{cl} = \frac{1}{M_{cr}}\sum_{i\in cr}M_i\dot{u}_{ik}^{cl}$$
$$\dot{u}_{sys,k}^{cl} = \frac{1}{(M_T-M_{cr})}\sum_{i\in cr}M_i\dot{u}_{ik}^{cl}$$

And the variables introduced in equation (3.3) are defined as follows;

UEP sensitivity coefficient

$$u_{ik}^u=\frac{\partial\theta_i^u}{\partial P_{mk}}=\lim_{\Delta P_{mk}\to 0}\frac{\theta_i^u(P_{mk}+\Delta P_{mk})-\theta_i^u(P_{mk})}{\Delta P_{mk}} \qquad (3.4)$$

where

$\theta_i^u(P_{mk}+\Delta P_{mk})$ ; i-th component of the controlling UEP for the perturbed power flow,

$\theta_i^u(P_{mk})$ ; i-th component of the controlling UEP for the base power flow case,

Physically $u_{ik}^u$ represents the ratio of i-th machine UEP angle change to k-th machine generation increase as the amount of generation increase goes to zero.

Clearing angle sensitivity coefficient

$$u_{ik}^{cl} = \frac{\partial \theta_i^{cl}}{\partial P_{mk}} = \lim_{\Delta P_{mk} \to 0} \frac{\theta_i^{cl}(P_{mk} + \Delta P_{mk}) - \theta_i^{cl}(P_{mk})}{\Delta P_{mk}} \tag{3.5}$$

where

$\theta_i^{cl}(P_{mk} + \Delta P_{mk})$ ; i-th machine angular position at fault clearing for the perturbed power flow,

$\theta_i^{cl}(P_{mk})$ ; i-th machine angular speed at fault clearing for the base power flow case,

Physically $u_{ik}^{cl}$ represents the ratio of i-th machine clearing angle change to k-th machine generation increase as the amount of generation increase goes to zero. The clearing angle is the angle at fault clearing time.

Clearing speed sensitivity coefficient

$$\dot{u}_{ik}^{cl} = \frac{\partial \tilde{\omega}_i^{cl}}{\partial P_{mk}} = \frac{d}{dt}\frac{\partial \theta_i^{cl}}{\partial P_{mk}} = \lim_{\Delta P_{mk} \to 0} \frac{\tilde{\omega}_i^{cl}(P_{mk} + \Delta P_{mk}) - \tilde{\omega}_i^{cl}(P_{mk})}{\Delta P_{mk}} \tag{3.6}$$

where

$\tilde{\omega}_i^{cl}(P_{mk} + \Delta P_{mk})$ ; i-th machine angular position at fault clearing for the perturbed power flow,

$\tilde{\omega}_i^{cl}(P_{mk})$ ; i-th machine angular speed at fault clearing for the base power flow case,

Physically $\dot{u}_{ik}^{cl}$ represents the ratio of i-th machine clearing speed change to k-th machine generation increase as the amount of generation increase goes to zero.

Observing equation (3.2) through (3.6) we can obtain $\Delta(\Delta V)$ caused by $\Delta P_{mk}$ as a linear combination of $\Delta P_{mk}$ if we know the value of the $\frac{\partial |E_i|}{\partial P_{mk}}$ and UEP angle, clearing angle and clearing speed sensitivity coefficients.

### 3.3 Derivation of Dynamic Sensitivity Equation

Generally speaking the trajectories of the power system during the faulted period are changing smoothly with respect to mechanical power input variation. Therefore they are differentiable with respect to mechanical input power at the machine terminals. The swing equations during the faulted period are given by, [1]

[1] The swing equation in COI reference frame is solved only for $i = 1, 2, ..., n-1$ , n-th machine angle and speed can be obtained from COI constraint equation (2.7).

$$M_i\dot{\omega}_i = P_i - P_{ei} - \frac{M_i}{M_T}P_{COI}\ , \quad i = 1, ..., n-1 \tag{3.7}$$

where

$$\begin{aligned} P_{ei} &= \sum_{\substack{j=1 \\ j\neq i}}^{n} [C_{ij}^f \sin\theta_{ij} + D_{ij}^f \cos\theta_{ij}] \\ P_i &= P_{mi} - E_i^2 G_{ii}^f \\ P_{COI} &= \sum_{i=1}^{n} P_i - \sum_{i=1}^{n}\sum_{\substack{j=1 \\ j\neq i}}^{n} D_{ij}^f \cos\theta_{ij} \\ M_T &= \sum M_i \end{aligned}$$

and

$C_{ij}^f = |E_i||E_j|B_{ij}^f$

$D_{ij}^f = |E_i||E_j|G_{ij}^f$

$G_{ij}^f$ ; real part of ij element of internal node reduced faulted admittance matrix.

$B_{ij}^f$ ; imaginary part of ij element of internal node reduced faulted admittance matrix.

Differentiating equation (3.7) with respect to $P_{mk}$ and rearranging we have the following set of differential equations, for some k( $1 \le k \le N$ ),

$$M_i\frac{d^2}{dt^2}(u_{ik}) = -Q_{ik}^f + \sum_{j=1}^{n} A_{ij}^f u_{jk}, \qquad i = 1, ..., n-1 \tag{3.8}$$

where

$$u_{ik} = \frac{\partial\theta_i}{\partial P_{mk}}$$

and

$$
\begin{aligned}
A^f_{ii} &= (1 - \frac{2M_i}{M_T}) \sum_{\substack{j=1 \\ j \neq i}}^{n} D^f_{ij} \sin\theta_{ij} - \sum_{\substack{j=1 \\ j \neq i}}^{n} C^f_{ij} \cos\theta_{ij} \\
A^f_{ij} &= C^f_{ij} \cos\theta_{ij} - D^f_{ij} \sin\theta_{ij} + \frac{2M_i}{M_T} \sum_{\substack{l=1 \\ l \neq j}}^{n} D^f_{lj} \sin\theta_{lj} \\
Q^f_{ik} &= \frac{M_i}{M_T} - \delta_{ik} \\
&\quad + \sum_{j=1}^{n} (\frac{\partial |E_i|}{\partial P_{mk}} |E_j| + |E_i| \frac{\partial |E_j|}{\partial Pmk})(B^f_{ij} \sin\theta_{ij} + G^f_{ij} \cos\theta_{ij}) \\
&\quad - \frac{M_i}{M_T} \sum_{j=1}^{n} \sum_{l=1}^{n} (\frac{\partial |E_l|}{\partial P_{mk}} |E_j| + |E_l| \frac{\partial |E_j|}{\partial P_{mk}}) G^f_{lj} \cos\theta_{lj} \\
{\delta_{ik}}^2 &= Kronecker\delta
\end{aligned}
$$

Observing the above dynamic sensitivity equation, we see that it is a time-varying second order linear differential equation which can be solved numerically if the appropriate initial conditions are given. To determine the coefficients of the dynamic sensitivity equation we have to know $\frac{\partial |E_i|}{\partial P_{mk}}$ and machine angle $\theta_i$ during the faulted period to determine the coefficients of dynamic sensitivity equation. The procedure to approximate $\frac{\partial |E_i|}{\partial P_{mk}}$ is given in Appendix A.

[2]It should not be confused with machine angle in fixed reference frame $\delta_i$ appeared in equation(2.1), and the position of center of inertia $\delta_0$ . Double subscripted $\delta$ is always kronecker delta.

$\delta_{ik} = 1$ for $i = k$,
$\delta_{ik} = 0$ for $i \neq k$ .

### 3.4 SEP Sensitivity and Controlling UEP Sensitivity

In the pre-fault system, if there is a generation change, internal voltage angles of synchronous machine in COI coordinate will change. In general the functional relation between generation change and angle changes is non-linear. If the amount of generation change is small enough, then it can be linearized.

Since we have the dynamic sensitivity equation, stable equilibrium point(SEP) sensitivity $\frac{\partial \theta_i^s}{\partial P_{mk}}$ which is static in nature, can be obtained easily by suppressing the dynamic term $M_i \frac{d^2}{dt^2}(u_{ik})$ from dynamic sensitivity equation (3.8). SEP sensitivity coefficients constitute the part of the required initial conditions for dynamic sensitivity equations to obtain the clearing angle and speed sensitivities.

Since the system is in the pre-fault state, the pre-fault value of network parameters should be used in the equations.

Using above procedure we can obtain (n-1) linear equations for n unknown variables for some k($1 \leq k \leq N$).

$$\sum_{j=1}^{n} A_{ij}^{pr} u_{jk}^{s} = Q_{ik}, \qquad i = 1, ..., n-1 \tag{3.9}$$

where

$$u_{jk}^{s} = \frac{\partial \theta_j^s}{\partial P_{mk}}$$

and

$$
\begin{aligned}
A_{ii}^{pr} &= (1-\frac{2M_i}{M_T})\sum_{\substack{j=1\\ j\neq i}}^{n} D_{ij}^{pr}\sin\theta_{ij}^{s} - \sum_{\substack{j=1\\ j\neq i}}^{n} C_{ij}^{pr}\cos\theta_{ij}^{s} \\
A_{ij}^{pr} &= C_{ij}^{pr}\cos\theta_{ij}^{s} - D_{ij}^{pr}\sin\theta_{ij}^{s} + \frac{2M_i}{M_T}\sum_{\substack{l=1\\ l\neq j}}^{n} D_{lj}^{pr}\sin\theta_{lj}^{s} \\
Q_{ik}^{pr} &= \frac{M_i}{M_T} - \delta_{ik} \\
&+ \sum_{\substack{j=1\\ j\neq i}}^{n}(\frac{\partial|E_i|}{\partial P_{mk}}|E_j| + |E_i|\frac{\partial|E_j|}{\partial Pmk})(B_{ij}^{pr}\sin\theta_{ij}^{s} + G_{ij}^{pr}\cos\theta_{ij}^{s}) \\
&-\frac{M_i}{M_T}\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{l=1}^{n}(\frac{\partial|E_l|}{\partial P_{mk}}|E_j| + |E_l|\frac{\partial|E_j|}{\partial P_{mk}})G_{lj}^{pr}\cos\theta_{lj}^{s} \\
&\text{for } i = 1,2,...,n-1 \text{ and } \quad k = 1,2,...,N
\end{aligned}
$$

and

$C_{ij}^{pr} = |E_i||E_j|B_{ij}^{pr}$

$D_{ij}^{pr} = |E_i||E_j|G_{ij}^{pr}$

$G_{ij}^{pr}$ ; real part of ij element of internal node reduced pre-fault admittance matrix

$B_{ij}^{pr}$ ; imaginary part of ij element of internal node reduced pre-fault admittance

Only (n-1) equations in n unknown variables are available, one more equation is needed to obtain a unique solution. The n-th equation can be obtained from COI coordinate constraint. COI constraint for sensitivity coefficient equation(3.12) and

(3.13) can be obtained by differentiating equation(3.10) and (3.11) respectively.

$$\sum_{i=1}^{n} M_i \theta_i = 0, \tag{3.10}$$

$$\sum_{i=1}^{n} M_i \tilde{\omega}_i = 0, \tag{3.11}$$

$$\sum_{i=1}^{n} M_i \frac{\partial \theta_i}{\partial P_{mk}} = 0, \tag{3.12}$$

$$\sum_{i=1}^{n} M_i \frac{\partial \tilde{\omega}_i}{\partial P_{mk}} = 0. \tag{3.13}$$

With equation(3.12) we can obtain following N sets of linear system.

$$\left(\hat{A}^{pr}\right)\left(\underline{u}_k^s\right) = \left(\underline{\hat{Q}}_k^{pr}\right) \tag{3.14}$$

for $k = 1, 2, ..., N$

where

$$\left(\hat{A}^{pr}\right) = \begin{pmatrix} A^{pr} \\ --- \\ \underline{M} \end{pmatrix} \tag{3.15}$$

and

$A^{pr}$ ; (n-1)x(n) matrix whose elements are defined in equation (3.9),

$$\underline{M} = (M_1\ M_2\ ...\ M_n),$$
$$\underline{u}_k^s = (u_{1k}^s\ u_{2k}^s\ \cdots\ u_{nk}^s)^t,$$ [3]
$$\underline{Q}_k^{pr} = (Q_{1k}^{pr}\ Q_{2k}^{pr}\ \cdots\ Q_{nk}^{pr})^t.$$

The UEP sensitivity equation can be obtained by modifying the SEP sensitivity equation(3.14). If we change every superscript 's' to 'u' and delete every superscript 'pr', then we can obtain the UEP sensitivity equation.

$$\left(\hat{A}\right)\left(\underline{u}_k^u\right) = \left(\hat{\underline{Q}}_k\right) \tag{3.16}$$

for $k = 1, 2, ..., N$

where

$$\left(\hat{A}\right) = \begin{pmatrix} A \\ --- \\ \underline{M} \end{pmatrix} \tag{3.17}$$

and

$A$ ; (n-1)x(n) matrix whose elements are defined below,

$\underline{u}_k^u = (u_{1k}^u\ u_{2k}^u\ \cdots\ u_{n,k}^u)^t,$

[3] t denotes the matrix transpose.

$$\underline{\hat{Q}}_k = (Q_{1k}\ Q_{2k} \cdots Q_{nk}\ \ 0\ )^t .$$

and

$$\begin{aligned}
A_{ii} &= (1 - \frac{2M_i}{M_T}) \sum_{\substack{j=1 \\ j\neq i}}^{n} D_{ij} \sin\theta_{ij}^u - \sum_{\substack{j=1 \\ j\neq i}}^{n} C_{ij} \cos\theta_{ij}^u \\
A_{ij} &= C_{ij}\cos\theta_{ij}^u - D_{ij}\sin\theta_{ij}^u + \frac{2M_i}{M_T} \sum_{\substack{l=1 \\ l\neq j}}^{n} D_{lj}\sin\theta_{lj}^u \\
Q_{ik} &= \frac{M_i}{M_T} - \delta_{ik} \\
&\quad + \sum_{\substack{j=1 \\ j\neq i}}^{n} (\frac{\partial|E_i|}{\partial P_{mk}}|E_j| + |E_i|\frac{\partial|E_j|}{\partial Pmk})(B_{ij}\sin\theta_{ij}^u + G_{ij}\cos\theta_{ij}^u) \\
&\quad - \frac{M_i}{M_T} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{l=1}^{n} (\frac{\partial|E_l|}{\partial P_{mk}}|E_j| + |E_l|\frac{\partial|E_j|}{\partial P_{mk}}) G_{lj}\cos\theta_{lj}^u
\end{aligned}$$

for $i = 1, 2, ..., n-1$ and $k = 1, 2, ..., N$

## 3.5 Key Procedure for Sensitivity Analysis

### 3.5.1 Solution of the sensitivity equation

The dynamic sensitivity equation which is a system of ordinary differential equation with time varying coefficients can be solved numerically if we are given the initial conditions. Just before and after the fault, the angular positions and speeds of the machines remain the same because of the inertia of the machines. As a result we have the following initial conditions.

$$\begin{aligned} u_{ik}(0) &= u^s_{ik} \\ \dot{u}_{ik}(0) &= 0 \;\; for\; i = 1, 2, ..., n \;\; k = 1, 2, ..., N-1 \end{aligned} \tag{3.18}$$

(3.19)

Dynamic sensitivity equations with above initial conditions are integrated until the fault is cleared, and we can obtain sensitivities of clearing angles and speeds.

$$\begin{aligned} u^{cl}_{ik} &= u_{ik}(t_{cl}) \\ \dot{u}^{cl}_{ik} &= \dot{u}_{ik}(t_{cl}) \;\; for\; i = 1, 2, ..., n \;\; k = 1, 2, ..., N-1 \end{aligned} \tag{3.20}$$

(3.21)

where

$t_{cl}$ ; clearing time

Solution of the SEP sensitivity equation requires solution of N sets of a system of linear equations. Because these N sets of linear system have the same coefficient matrix $\hat{A}^{pr}$ with different vector $\underline{\hat{Q}}^{pr}_k$ corresponding to k, they can be solved simultaneously by using the Gaussian elimination method. The UEP sensitivity equations can be solved in the same way.

### 3.5.2 Procedure to obtain the energy margin sensitivity with respect to generation shifts

Sensitivity of the energy margin with respect to generation shifts is possible on the premise that base power flow case energy margin is accurately assessed by the TEF method. In the TEF program, [4] a series of input data are needed. In the process of calculating the energy margin of base power flow case, a lot of intermediate results are obtained. Among those data, the following data are essential to sensitivity program.

- Machine dynamic data ($X'_d$ and $M_i$)
- Base power flow solutions
- Disturbance data (fault type, fault duration and control action to clear fault)
- Network information (internal node reduced pre-fault, faulted and post-fault bus admittance matrix )
- Pre-fault SEP and post-fault controlling UEP angles
- Fault trajectory (angle and speed of machines during fault period)[5]

The key procedure for estimating sensitivity of the energy margin with respect to generation shift is as follows (see Figure 3.1).

[4] "DIRECT" a program distributed by the Electric Power Research Institute Software Center (developed by Ontario Hydro and Iowa State University).

[5] In the TEF program to obtain the clearing angles and speeds of machines, the swing equation is integrated during fault period using an approximate technique.

step1 ; Obtain data and intermediate results from the TEF program.

Read generation shift data.

step2 ; Calculate $\frac{\partial |E_i|}{\partial P_{mk}}$ using the approximation (see Appendix A).

step3 ; Obtain the SEP sensitivity equations $\left(\hat{A}^{pr}\right)\left(\underline{u}_k^s\right) = \left(\underline{\hat{Q}}_k^{pr}\right)$ and solve for $\underline{u}_k^s$ for k=1,2,...,N-1.

step4 ; Obtain the dynamic sensitivity equations

$$M_i \frac{d^2}{dt^2}(u_{ik}) = -Q_{ik}^f + \Sigma_{j=1}^n A_{ij}^f u_{jk}$$

for $i = 1, ..., n-1$ and $k = 1, ..., N$

and solve for $u_{ik}^{cl}$ .

step5 ; Obtain the UEP sensitivity equations $\left(\hat{A}\right)\left(\underline{u}_k^u\right) = \left(\underline{\hat{Q}}_k\right)$ and solve for $\underline{u}_k^u$ for k=1,2,...,N-1.

step6 ; Calculate the energy margin sensitivity with respect to generation shifts $\frac{\partial(\Delta V)}{\partial P_{mk}}$ for k=1,2,...,N-1.

step7 ; Calculate the new energy margin $(\Delta V)_{new}$

$$(\Delta V)_{new} = (\Delta V)_{old} + \sum_{k=1}^{N} \frac{\partial(\Delta V)}{\partial P_{mk}} \delta P_{mk}$$

As explained before, the coefficients of the dynamic sensitivity equation are time-varying. In the TEF program for the base power flow case, fault trajectory is obtained by integrating the swing equations. To save the memory storage for fault trajectory, dynamic sensitivity equations are integrated at the same time when the swing equations are integrated.

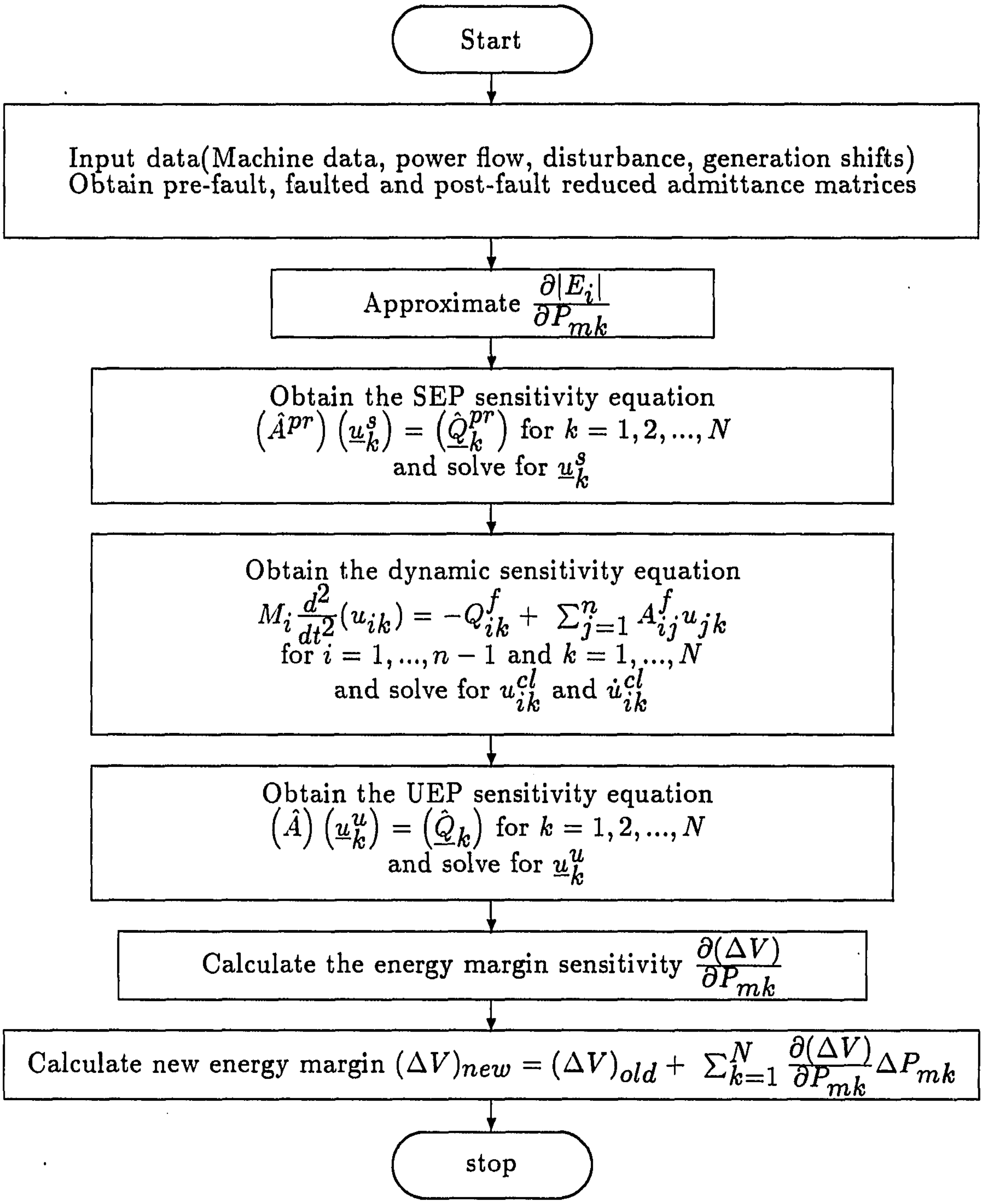


Figure 3.1: Flow Chart of Sensitivity Program

### 3.5.3 Computational burden of sensitivity program

The computational work required in the proposed sensitivity technique is compared with the work involved in repetitive application of the TEF program and power flow program.

To determine the critical generator loading limit using the TEF program, several runs of the TEF program and power flow program are needed.

With the proposed sensitivity technique the behavior of energy margin change ( by equation (3.2)) can be obtained by running the sensitivity program once. The sensitivity program is developed by adding several subroutines and modifying the TEF program. The additional computational requirement of the sensitivity program is mainly due to the following :

1. Integrating N sets of the dynamic sensitivity equations whose order and the dimension of the variable are the same as the swing equations.

2. Solving the two linear system of equations to obtain SEP and UEP sensitivity coefficients.

Assuming that computational burden of integrating the swing equations and integrating the dynamic sensitivity equation are approximately equal, the ratio of computational work of the proposed sensitivity technique to that of TEF program involved in integrating the differential equations $R_W$ are given as follows.

$$R_W = \frac{N}{2(N-1)} \quad (N > 1)$$

It should be noted that this comparison is made neglecting the work required to obtain $2(N-1)$ power flow solutions in order to run TEF program to obtain the sensitivity information. In the proposed sensitivity technique the corresponding work to obtain $2(N-1)$ power flow solutions consists of the solution to two system of linear algebraic equations. It is obvious that the latter is far less than the former.

# 4 SENSITIVITY OF THE ENERGY MARGIN WITH RESPECT TO TRANSMISSION INTERFACE POWER FLOW CHANGES

## 4.1 Determination of Transmission Interface Power Flow Limits under Stability Constrained Situation

In operating a stability limited power system with limited margin for secure operation, it is essential for the operator to obtain limits for a variety of conditions and contingencies. These limits are mainly obtained in terms of plant generation or transmission interface power flow limits. In certain situations when transmission interface power flow changes are made for economic reasons, it is desirable to determine the effect on the stability of the system for different scenarios in order to suitably plan for preventive action.

When the system is radial, linear sensitivity analysis is sufficient to determine interface power flow limits. To determine the line power flow limits in an interconnected system, however, the analysis is not as straight forward as that for determining the plant generator loading limits.

For a given transmission interface power flow change, the energy margin can not be uniquely determined. This is because a given transmission interface power flow change can be obtained by using different combinations of generation which will result in a different amount of energy margin change.

For a particular contingency, the change of energy margin with respect to a generation shift[1] can be obtained using the assumption that the total generation remains constant and that the energy margin varies linearly with respect to the generation shifts.

$$\Delta(\Delta V) = \sum_{k=1}^{N-1} \alpha_k \Delta P_{mk} \tag{4.1}$$

where

$\alpha_k = \frac{\partial(\Delta V)}{\partial P_{mk}} - \frac{\partial(\Delta V)}{\partial P_{mN}} \quad k = 1, 2, ..., N-1$ ,

N-1 = Number of generator at which generation can be shifted with respect to the reference generator( N-th machine is the reference generator)

A linear relationship also exists between a line power flow change and generation shifts [32, chapter 11] given by

$$\Delta I_l = \sum_{k=1}^{N-1} S_{lk} \Delta P_{mk} \tag{4.2}$$

where

$S_{lk}$ = Distribution factor of line $l$ due to generation shift at generator $k$, $k = 1, 2, ..., N-1$

Using the relationship shown in equation (4.1) and (4.2) a procedure to relate the change in interface power flow for a specific change in the base case margin will be developed. In doing so the ranges of transmission interface power flow limits

[1]It is assumed that the change of generation $\Delta P_{mk}$ at k-th machine is compensated by an opposite change of generation at a reference machine.

are obtained as a result of generation shifts. In this development the following assumption are made.

1. The critical generators at which generation can be shifted with respect to the reference machine are known.

2. When generation is shifted at the critical generators, the sensitivity of the energy margin to the change at these generators $\alpha_k$, $k = 1, 2, ..., N-1$ have the same sign. The distribution factors for a line $l$ ;$S_{lk}$ $k = 1, 2, ..., N-1$ with respect to the critical generators should all have the same sign in order not to cancel out the power flow changes caused from generation shifts at the critical generators. On a critical line this is normally the case.

3. The range of allowed generation shifts at each generator is of such a value as to result in zero energy margin. If the system is stable(or unstable) for the base power flow case, generation shifts considered will result in zero energy margin.

4. The range of generation shifts assumed in 3 do not exceed the machine or equipment ratings.

Then our problem takes the following form,

" For a particular contingency determine the minimum and maximum values of interface power flow limits satisfying the given constraints."

If the above problem is reformulated mathematically, we then have , Minimize or Maximize

$$\Delta I_l = \sum_{k=1}^{N-1} S_{lk} \Delta P_{mk}$$

Subject to

$$\sum_{k=1}^{N-1} \alpha_k \Delta P_{mk} = -\Delta V$$

and

$$-\Delta V \le \alpha_k \Delta P_{mk} \le 0 \quad , \quad if \quad \Delta V > 0$$

$$0 \le \alpha_k \Delta P_{mk} \le -\Delta V \quad , \quad if \quad \Delta V > 0$$

$$for \quad k = 1, 2, ..., N-1$$

where

$\Delta V$ ; system energy margin which has to be adjusted to result in zero energy margin for the stable or unstable case

If we transform the equation (4.1) using

$$\Delta P'_{mk} = \alpha_k \Delta P_{mk}, \quad k = 1, 2, ..., N-1 \tag{4.3}$$

and define $a_k$ by

$$a_k = \frac{S_{lk}}{\alpha_k}, \quad k = 1, 2, ..., N-1 \tag{4.4}$$

then our problem becomes,

Minimize or maximize

$$\Delta I_l = \sum_{k=1}^{N-1} a_i \Delta P'_{mk} \tag{4.5}$$

Subject to

$$\sum_{k=1}^{N-1} \Delta P'_{mk} = -\Delta V \tag{4.6}$$

and

$$\begin{aligned} -\Delta V \leq \Delta P'_{mk} \leq 0, \quad if \quad \Delta V > 0 \\ 0 \leq \Delta P'_{mk} \leq -\Delta V, \quad if \quad \Delta V < 0 \\ for \quad k = 1, 2, ..., N-1 \end{aligned} \tag{4.7}$$

The above problem is a linear programming problem. Because of the simplicity of the objective function and constraint equations, the solution can be easily obtained.

Define the generation shift vector as $\Delta \underline{P}_m$

$$\Delta \underline{P}_m = (\Delta P_{m1},\ \Delta P_{m2},\ ...,\ \Delta P_{m,N-1}) \tag{4.8}$$

using the transforming equation (4.3) we have

$$\Delta \underline{P'}_m = (\Delta P'_{m1},\ \Delta P'_{m2},\ ...,\ \Delta P'_{m,N-1}) \tag{4.9}$$

Using the theorem of the simplex method [33, chapter 4] the minimum and maximum amount of line power flow change $\Delta I_{l*}$ and $\Delta I_l^*$ are respectively given by( Proof is given in Appendix B),

$$\Delta I_{l*} = -\Delta V \; a_{i*} \tag{4.10}$$

$$\text{when } \Delta \underline{P}'_m = -\Delta V \; e_i$$ [2]

$$\Delta I_l^* = -\Delta V \; a_j^* \tag{4.11}$$

$$\text{when } \Delta \underline{P}'_m = -\Delta V \; e_j$$

where

$$|a_{i*}| = min \; \{|a_i|\}$$

$$|a_j^*| = max \; \{|a_j|\}$$

## 4.2 Sensitivity of the Energy Margin with respect to Transmission Interface Power Flow Changes

In the foregoing section transmission interface power flow limits were obtained assuming linear behavior of the energy margin with respect to line power flow changes. If the system energy margin is not small enough to justify linear analysis, then repetitive application of the TEF program and linear analysis can give the transmission interface power flow limits.

[2] $e_i$ is a $(N-1)$ dimensional unit vector whose i-th component is 1 and 0 for other components

Define sensitivity of the energy margin with respect to the transmission interface power flow change as $\beta_l$ ,

$$\beta_l = \frac{\Delta(\Delta V)}{\Delta I_l} \tag{4.12}$$

As explained in the foregoing section, $\beta_l$ does not have an unique value. But we can obtain the maximum or minimum value of $|\beta_l|$ assuming the following.

1. The critical generators at which generation can be shifted with respect to the reference machine are known.

2. $S_{lk}$ has the same sign for all $k = 1, 2, ..., N-1$ .

3. The range of generation shifts at each generator is of such a value as to result in the given line power flow change.

4. The range of generation shifts assumed in 3 do not exceed the machine or equipment ratings.

Then the problem to obtain the range of $|\beta_l|$ becomes the followings.

Minimize or maximize

$$\Delta(\Delta V) = \sum_{k=1}^{N-1} \alpha_k \Delta P'_k \tag{4.13}$$

Subject to

$$\sum_{k=1}^{N-1} \Delta S_{lk} P_{mk} = \Delta I_l \tag{4.14}$$

and

$$-\Delta I_l \leq S_{lk}\Delta P'_{mk} \leq 0, \quad if \quad S_{lk}\Delta P_{mk} < 0$$
$$0 \leq S_{lK}\Delta P_{mk} \leq \Delta I_l, \quad if \quad S_{lk}\Delta P_{mk} > 0 \qquad (4.15)$$
$$for \quad k = 1, 2, ..., N-1$$

Then the maximum and minimum amount of energy margin change $\Delta(\Delta V)^*$ and $\Delta(\Delta V)_*$ are given respectively,

$$\Delta(\Delta V)^* = \frac{\Delta I_l}{a_{i*}} \qquad (4.16)$$
$$\text{when } \Delta \underline{P}_m = \frac{\Delta I_l}{a_{i*}\alpha_i} e_i$$

$$\Delta(\Delta V)_* = \frac{\Delta I_l}{a_j^*} \qquad (4.17)$$
$$\text{when } \Delta \underline{P}_m = \frac{\Delta I_l}{a_j^*\alpha_j} e_j$$

From the equation (4.16) and (4.17) the minimum and maximum value of sensitivity of the energy margin with respect to the transmission interface power flow change $|\beta_l|$ are given respectively.

$$\beta_l^* = \quad max \quad \{|\beta_l|\} = \frac{\Delta(\Delta V)^*}{\Delta I_l} \qquad (4.18)$$
$$\beta_{l*} = \quad min \quad \{|\beta_l|\} = \frac{\Delta(\Delta V)_*}{\Delta I_l} \qquad (4.19)$$

$\beta_l$ represents the ratio of the energy margin change to the transmission interface power flow change.

## 4.3 Remarks on the Constraints

In Sections 4.1 and 4.2 several assumptions are made. All assumptions except the same sign constraint are necessary to make a meaningful linear programming problem. Because of the sign constraint and simple inequality constraints the solutions were obtained easily without using the standard algorithm for solving the linear programming problem.

To monitor line power flow limits, broad range of line power flow limits are not desirable. For a particular contingency we can pick an appropriate line which shows a narrow range of the transmission interface power flow limits.

1. For the assumed generation shifts all the $a_k$ should be of the same sign.
2. The difference between $\Delta I_{l*}$ and $\Delta I_l^*$ should be small.
3. For the assumed generation shifts the distribution factors of the line should be large.

Rule 1 generally holds for the critical lines. Since a small distribution factor results in small line power flow changes due to generation shifts, lines having small distribution factors with respect to a certain generator will not have sufficient change in power flow due to change in generation. This also indicates that these lines may not be critical lines with respect to the generation changes considered. It is also obvious that even though a certain line shows good characteristics to monitor line power flow changes for a particular contingency, it can be an inappropriate line for monitoring power flow changes for other contingencies.

The three conditions stated above are well satisfied for a radial system. In an interconnected system, however, the above three conditions are satisfied only for a critical line which absorbs a major portion of power flow change due to generation shift between the critical generators and the reference generator.

# 5 THE TEST SYSTEM

## 5.1 Test System

The test system used in validation study is the Reduced Iowa System, which was developed by the Power System Computer Service at Iowa State University. Figure 5.1 shows the main study area.

The base power-flow system is a model of 862 buses and 1323 lines and transformers. Most of the transmission lines are 345 KV and 161 KV, while some of the lines are 230 KV, 115 KV, or 69 KV. A partial one-line diagram of the key buses and the major high voltage transmission lines in the area are shown in Figure 5.2. Load flow data are given in reference [16].

This base power-flow model was reduced by a network reduction program to a model with 163 buses and with 304 lines and transformers. The resulting Reduced Iowa System is shown in Figure 5.3. The generator dynamic data, together with initial operation conditions are given in Table 5.1.

This test system was used to simulate faults primarily in the western part of the network along the Missouri river. Several generating plants are located electrically close to each other. A disturbance in that part of the network substantially influences the motion of several generators and very complex Modes of Disturbance can occur.

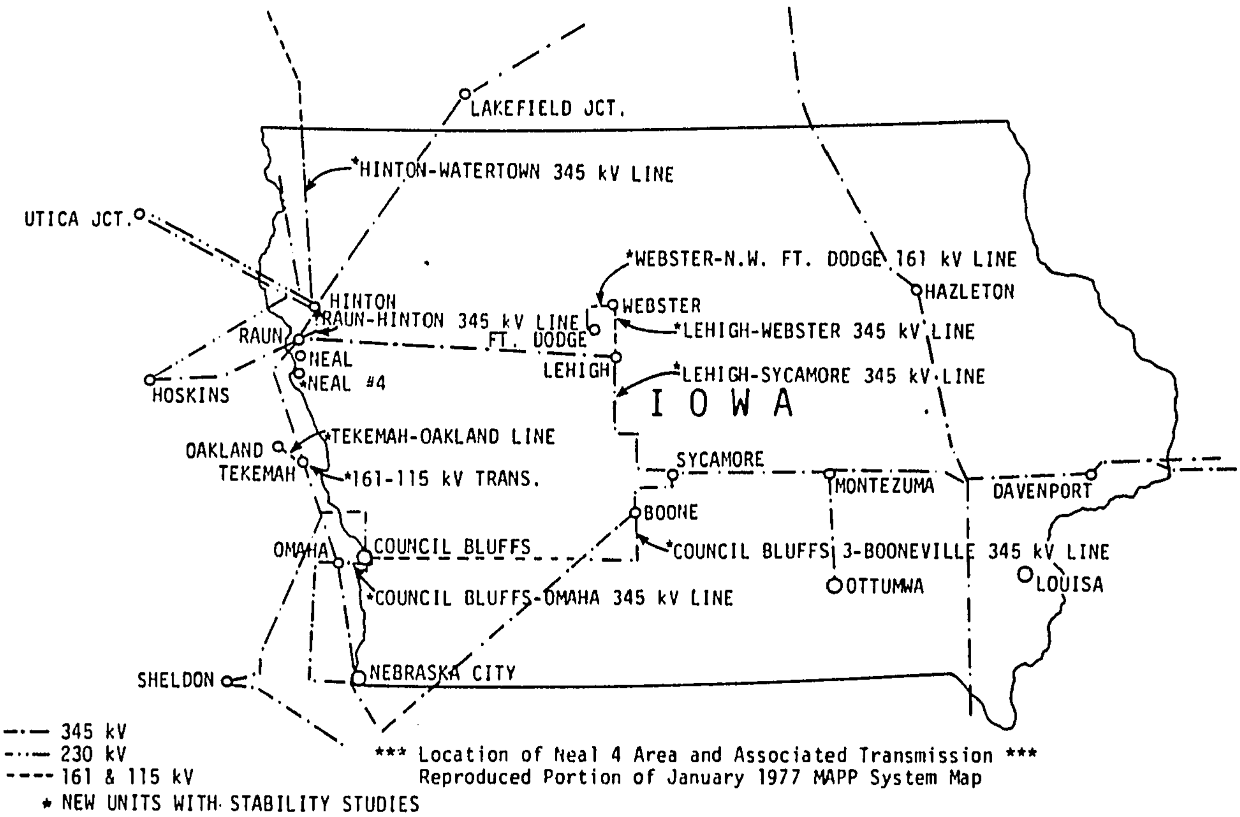


Figure 5.1: The Main Study Region for the Reduced Iowa System

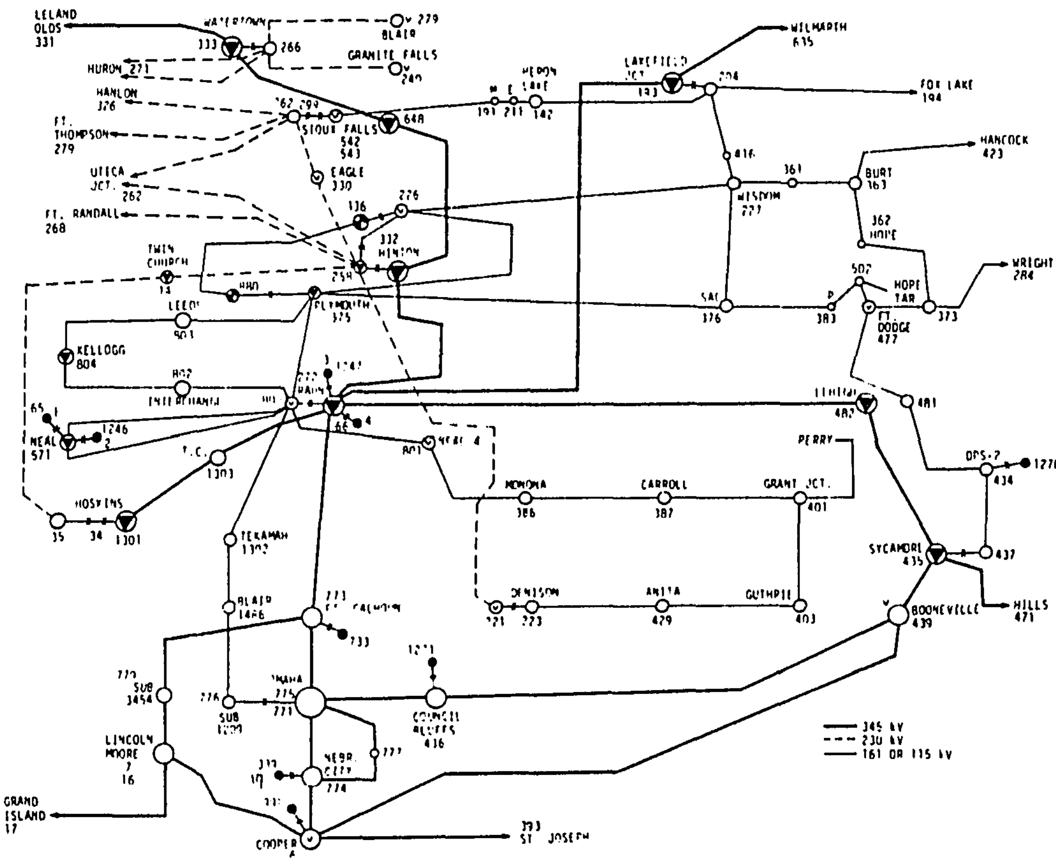


Figure 5.2: One-Line Diagram of the Study Area

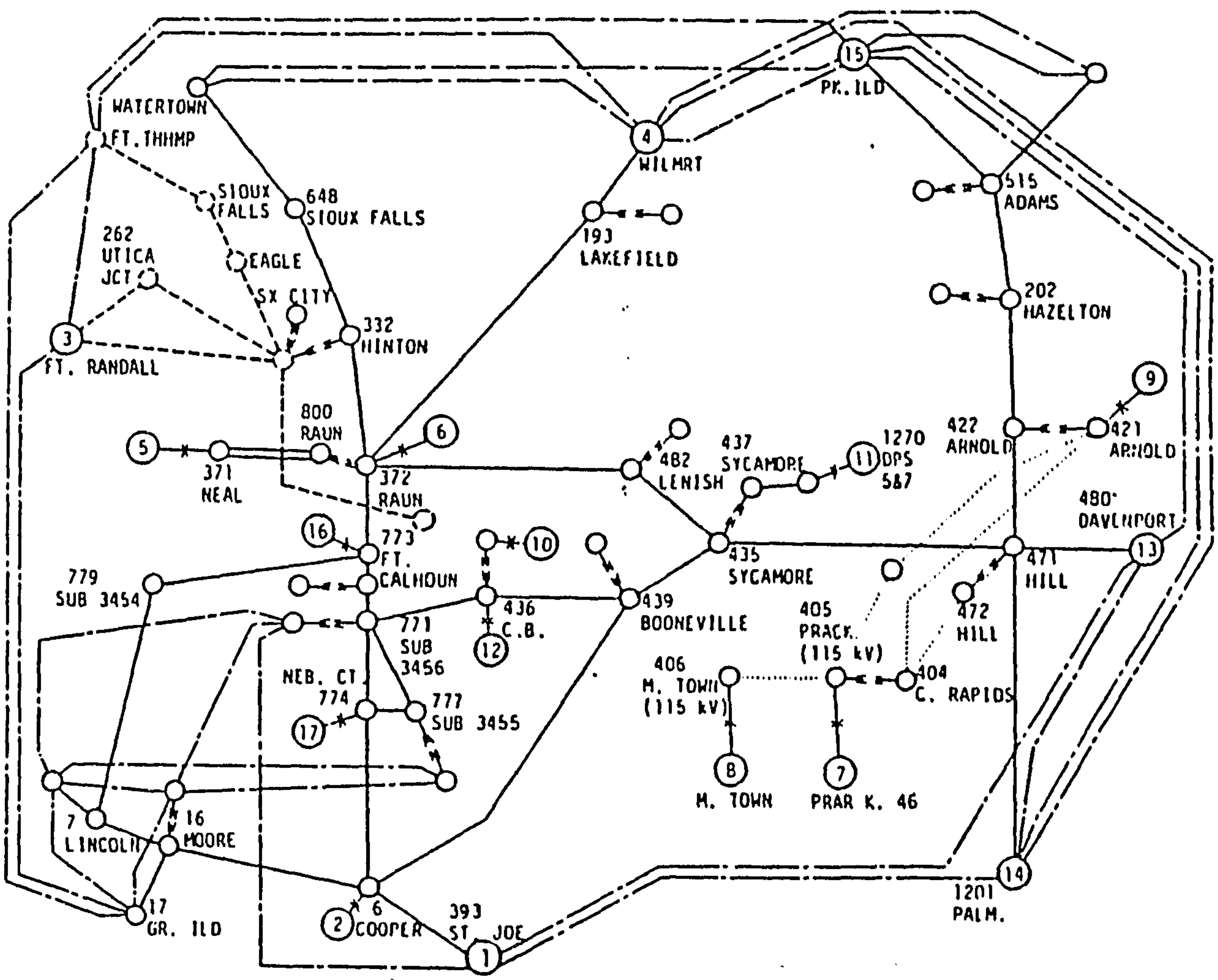


Figure 5.3: 17-Generator System (Reduced Iowa System)

Table 5.1: Generator Data and Initial Conditions of the 17-Generator System

| | Generator Parameters[a] | | Initial Conditions | | |
|---|---|---|---|---|---|
| | | | Power | Internal Voltage | |
| Generator Number | H (MW/MVA) | $x'_d$ (pu) | $Pmo^a$ (pu) | E (pu) | $\delta$ (degree) |
| 1 | 100.00 | 0.0040 | 20.000 | 1.00319 | -27.93 |
| 2 | 34.56 | 0.0437 | 7.940 | 1.13337 | -1.34 |
| 3 | 80.00 | 0.0100 | 15.000 | 1.03015 | -16.32 |
| 4 | 80.00 | 0.0050 | 15.000 | 1.00112 | -26.09 |
| 5 | 16.79 | 0.0507 | 4.470 | 1.06795 | -6.23 |
| 6 | 32.49 | 0.0206 | 10.550 | 1.05055 | -4.56 |
| 7 | 6.65 | 0.1131 | 1.309 | 1.01610 | -23.04 |
| 8 | 2.66 | 0.3115 | 0.820 | 1.12346 | -26.94 |
| 9 | 29.60 | 0.0535 | 5.519 | 1.11930 | -12.40 |
| 10 | 5.00 | 0.1770 | 1.310 | 1.06517 | -11.12 |
| 11 | 11.31 | 0.1049 | 1.730 | 1.07774 | -24.35 |
| 12 | 19.79 | 0.0297 | 6.200 | 1.06097 | -10.11 |
| 13 | 200.00 | 0.0020 | 25.709 | 1.01058 | -28.15 |
| 14 | 200.00 | 0.0020 | 23.875 | 1.02059 | -26.73 |
| 15 | 100.00 | 0.0040 | 24.670 | 1.01861 | -21.10 |
| 16 | 28.60 | 0.0559 | 4.550 | 1.12433 | -6.68 |
| 17 | 20.66 | 0.0544 | 5.750 | 1.11166 | -4.39 |

[a] On a 100-MVA base.

## 5.2 Test Cases

Contingency

Six contingencies were studied on the reduced Iowa System. The corresponding conditions of each contingency are listed in Table 5.2.

In Table 5.2 fault clearing time for test cases are longer than usual clearing time. These values of fault clearing times were chosen such that the energy margin of the base power flow case was small for the purpose of analysis.

Table 5.2: Condition of the Contingencies

| Faulted Bus( 3 phase fault ) | | Trip Line | | Fault Clearing | Energy Margin for |
|---|---|---|---|---|---|
| Name | Number | From | To | time (sec) | Base Power Flow |
| Ft.Cal. | 773 | 77 | 773 | 0.342 | 1.719995 |
| Ft.Cal. | 773 | 77 | 773 | 0.367 | -1.794749 |
| C.Blff. | 436 | 436 | 771 | 0.187 | 1.675387 |
| C.Blff. | 436 | 436 | 771 | 0.209 | -1.815234 |
| Cooper | 6 | 6 | 439 | 0.191 | 1.994042 |
| Cooper | 6 | 6 | 439 | 0.219 | -2.225264 |

Generation shifts

For each contingency, the generators at which generation is shifted are as follows($G_9$;reference machine). These generators are severely disturbed during disturbance and have large effect on the energy margin change.

1. Ft.Calhoun fault : $G_{16}$, $G_{12}$, $G_{17}$, ($G_{13}$ *and* $G_{17}$)

2. Council Bluff fault : $G_{10}$, $G_{12}$

3. Cooper fault : $G_2$

# 6 RESULTS

## 6.1 Determination of Critical Generator Loading Limits

The sensitivity program was tested to determine critical generator loading limits for the particular contingencies chosen.

### 6.1.1 Procedure

The test procedures for the sensitivity program are as follows;

Step1 ; Select contingency

Step2 ; Run the sensitivity program and determine critical generator loading limits assuming linear behavior of the energy margin change with respect to generation shifts.

Step3 ; Run the power flow program to obtain a modified power flow solution using generation shift data obtained in step2.

Step4 ; Run the TEF program using the modified power flow solution . If $(\Delta V)_{new} < \epsilon$ then stop, otherwise go to step 5.

Step5 ; Adjust generation shift using following equation,

$$\Delta P_{mk}^{ad} = \frac{(\Delta V)_{old}}{(\Delta V)_{old} - (\Delta V)_{new}} \Delta P_{mk}$$

where

$\Delta P_{mk}$ ; generation shift obtained at step 2

Step 6 ; If $|\Delta^{ad}_{mk}| < \epsilon$ then stop, otherwise run power flow program to obtain new power flow solution using adjusted generation shift obtained at step 5 and go to step 2.

### 6.1.2 Test results

For the base power flow case, the sensitivity program was run. Partial derivatives of the energy margin with respect to generation changes are given in Table 6.1.

Table 6.1: Partial Derivative of Energy Margin with respect to Generation Change

| Contingency | | Partial Derivative of Energy Margin with respect to Generation Change | | | | | |
|---|---|---|---|---|---|---|---|
| Faulted Bus | Clearing time(sec) | $\frac{\partial(\Delta V)}{\partial P_{m2}}$ | $\frac{\partial(\Delta V)}{\partial P_{m10}}$ | $\frac{\partial(\Delta V)}{\partial P_{m12}}$ | $\frac{\partial(\Delta V)}{\partial P_{m17}}$ | $\frac{\partial(\Delta V)}{\partial P_{m16}}$ | $\frac{\partial(\Delta V)}{\partial P_{m9}}$ |
| Ft.Cal. | 0.342 | -1.6709 | -2.8617 | -3.5424 | -2.4883 | -3.9551 | 0.5865 |
| Fault | 0.367 | -1.7072 | -3.0598 | -3.7676 | -2.6095 | -4.2572 | 0.6075 |
| C.Blff. | 0.187 | 0.06564 | -3.5155 | -4.3243 | -0.1309 | 0.06355 | 0.13882 |
| Fault | 0.209 | 0.07756 | -3.8470 | -4.9027 | -0.1843 | 0.07074 | 0.14778 |
| Cooper | 0.191 | -3.3397 | 0.02354 | -0.0854 | -0.2776 | 0.09629 | 0.12736 |
| Fault | 0.219 | -2.1366 | 0.08338 | 0.04989 | -0.02080 | 0.08704 | 0.10026 |

Using the test procedure generator loading limits were obtained and shown in Table 6.2. It should be noted that sufficiently accurate generator loading limits were obtained at step5 for all the cases considered.

Table 6.2: Generator Loading Limits($G_9$;Reference Machine)

| Contingency | | | Shifting Generator | Loading Limit Sensit.Anal. (MW) | Loading Limit Full TEF (MW) | Relative Error (%) |
|---|---|---|---|---|---|---|
| Fault Location | Clearing time(sec) | | | | | |
| Ft.Cal. Fault | 0.342 sec | Stable | $G_{16}$ | $\Delta P_{m16} = 37.87$ | $\Delta P_{m16} = 37.26$ | 1.6 |
| | | | $G_{12}$ | $\Delta P_{m12} = 41.66$ | $\Delta P_{m12} = 40.85$ | 2.0 |
| | | | $G_{17}$ | $\Delta P_{m17} = 55.94$ | $\Delta P_{m17} = 54.31$ | 3.0 |
| | | | $G_{17}$ | $\Delta P_{m17} = 56.03$ | $\Delta P_{m17} = 54.58$ | |
| | | | $G_{13}$ | $\Delta P_{m13} = -56.03$ | $\Delta P_{m13} = -54.58$ | -2.7 |
| | 0.367 | Unstable | $G_{16}$ | $\Delta P_{m16} = -36.89$ | $\Delta P_{m16} = -37.53$ | -1.7 |
| | | | $G_{12}$ | $\Delta P_{m12} = -41.02$ | $\Delta P_{m12} = -42.79$ | -4.1 |
| | | | $G_{17}$ | $\Delta P_{m17} = -55.79$ | $\Delta P_{m17} = -56.80$ | -1.8 |
| C. Blff. Fault | 0.187 | Stable | $G_{12}$ | $\Delta P_{m12} = 37.54$ | $\Delta P_{m12} = 37.33$ | 0.6 |
| | | | $G_{10}$ | $\Delta P_{m10} = 45.85$ | $\Delta P_{m10} = 45.88$ | -0.1 |
| | 0.209 | Unstable | $G_{10}$ | $\Delta P_{m10} = -45.44$ | $\Delta P_{m10} = -44.69$ | 1.7 |
| | | | $G_{12}$ | $\Delta P_{m12} = -35.94$ | $\Delta P_{m12} = -35.52$ | 1.2 |
| Cooper fault | 0.191 | Stable | $G_2$ | $\Delta P_{m2} = 57.51$ | $\Delta P_{m2} = 56.63$ | 1.6 |
| | 0.219 | Unstable | $G_2$ | $\Delta P_{m2} = -55.06$ | $\Delta P_{m2} = -54.65$ | 0.8 |

Generally speaking sensitivities of the energy margin with respect to generation change obtained from sensitivity program are accurate for the severely disturbed machines.

### 6.1.3 Accuracy of the sensitivity analysis

In the previous section results of sensitivity analysis are shown and its accuracy is well within the limits for practical application.

But if we look at generation loading limits, the differences from base power flow case are close to 40 MW for all cases considered. It is natural to ask what accuracy can we get in predicting large difference of generator loading from the base power flow case. To answer this question 7 cycle(0.1167 sec) faults are chosen to predict the energy margin change.

For these cases the transient energy margins are so large that generator loading hit the machine thermal rating well before it reaches generator stability loading limits. But for the purpose of analysis, approximately 300 MW generation shifts are considered neglecting generator thermal ratings.

With this large value of generation shift, however, the amount of generation shift is not enough to result in zero system energy margin. Instead of obtaining the generator loading limits, required generation shifts are predicted to obtain the predetermined system energy margin change.

Table 6.3: Energy Margin Change Behavior due to Large Generation Shifts

| Contingency | Full TEF Analysis | | | | | Sensitivity Analysis | Error (%) |
|---|---|---|---|---|---|---|---|
| | Base Power Flow | | Modified Power Flow | | Engy.Marg. | | |
| | Power(MW) | $\Delta V$ | Gen.Shift(MW) | $\Delta V$ | Chg. $\Delta(\Delta V)$ | Gen.Shift(MW) | |
| Ft.Cal.<br>Fault<br>$t_{cl}$ = 7cycle | $P_{m12} = 620$<br>$P_{m16} = 455$<br>$P_{m17} = 575$ | 27.0906 | $\Delta P_{m12} = 100$<br>$\Delta P_{m16} = 100$<br>$\Delta P_{m17} = 100$ | 20.8164 | -6.2742 | $\Delta P_{m12} = 94.08$<br>$\Delta P_{m16} = 94.08$<br>$\Delta P_{m17} = 94.08$ | -5.9 |
| C.Blff.<br>Fault<br>$t_{cl}$ = 7cycle | $P_{m10} = 131$<br>$P_{m12} = 620$ | 9.0392 | $\Delta P_{m10} = 150$<br>$\Delta P_{m12} = 150$ | 1.37461 | -7.6646 | $\Delta P_{m10} = 139.05$<br>$\Delta P_{m12} = 139.05$ | -7.3 |
| Cooper<br>Fault<br>$t_{cl}$ = 7cycle | $P_{m2} = 794$ | 8.8675 | $\Delta P_{m2} = 300$ | 2.3357 | -6.5318 | $\Delta P_{m2} = 292.00$ | -2.7 |

The nature of the faults are the same as for those shown in Table 6.1 except for the fault duration. For the Ft.Calhoun fault it is assumed that generation is shifted between three critical generators($G_{12}$, $G_{16}$, $G_{17}$) and the reference machine with increased generation at the critical generators equally distributed among the three critical generators. For the Council Bluff fault it is assumed that generation is shifted between two critical generator($G_{10}$, $G_{12}$) and the reference machine with increased generation at the critical generators equally distributed. For the Cooper fault only $G_2$ is shifting generation with respect to the reference machine. In Table 6.3 the results of sensitivity analysis are compared with those obtained from the TEF program and power flow studies.

Observing the errors in sensitivity analysis to determine generation shifts to obtain the given energy margin changes, the sensitivity analysis gives errors which are larger than those obtained in Table 6.2. However, these errors are within 10 % for the cases considered. These relatively small errors are mainly due to linear behavior of the energy margin change over a wide range with respect to generation change.

## 6.2 Determination of the Interface Power Flow Limits

Several key lines are chosen to monitor line power flow limits for the assumed contingencies. Distribution factors for these lines were obtained using the Philadelphia Electric Company(PECO) power flow program and are shown in Table 6.4.

From Table 6.4 line 471 Hill – 435 Sycamore is clearly a critical line for generation shifts at the all critical generators. When there are generation shifts between critical group of machines and reference machine(generator 9), the line 471(Hill)–

435(Sycamore) serves as a major passage for these generation shifts. This passage behaves like a radial system as far as the assumed generation shifts are considered.

Table 6.4: Distribution Factors of Monitored Line($G_9$:reference)

| | Distribution Factor | | | | |
|---|---|---|---|---|---|
| Monitored Line | $G_2$ | $G_{10}$ | $G_{12}$ | $G_{17}$ | $G_{16}$ |
| 471 Hill–435 Sycamore | -0.296 | -0.313 | -0.316 | -0.302 | -0.301 |
| 193 Lakefield – 372 Raun | -0.101 | -0.115 | -0.116 | -0.109 | -0.138 |
| 7 Lincoln – 779 Sub.3454 | 0.069 | -0.052 | -0.056 | 0.007 | -0.182 |
| 6 Cooper – 393 St.Joe | 0.266 | 0.159 | 0.176 | 0.226 | 0.174 |

Line 7 Lincoln – 779 Sub.3454 has a small distribution factor, 0.007, with respect to generation shift at $G_{17}$. As a result this line is not critical with respect to generation shift at generator $G_{17}$.

Table 6.5: Sensitivity of Energy Margin with respect to Generation Shift $\alpha_k$ ($G_9$:reference machine)

| | $\alpha_k = (\frac{\partial(\Delta V)}{\partial P_{mk}} - \frac{\partial(\Delta V)}{\partial P_{m9}})$ | | | | |
|---|---|---|---|---|---|
| Contingency | $G_2$ | $G_{10}$ | $G_{12}$ | $G_{17}$ | $G_{16}$ |
| Ft.Calhoun fault<br>$t_{cl} = 0.342$ sec<br>$\Delta V = 1.719995$ | -2.2573 | -3.4482 | -4.1288 | -3.0748 | -4.5416 |
| Council Bluff fault<br>$t_{cl} = 0.187$ sec<br>$\Delta V = 1.675388$ | -0.1004 | -3.6543 | -4.4631 | -0.3192 | -0.08627 |
| Cooper fault<br>$t_{cl} = 0.191$ sec<br>$\Delta V = 1.99404$ | -3.4671 | -0.1343 | -0.2427 | -0.4524 | -0.04053 |

For a particular contingency, line power flow limits were determined by the sensitivity analysis. These results were compared with those of repetitive run of the TEF program and power flow program. In table 6.5 values of $\alpha_k$ are shown. For the three contingencies considered earlier, line power flow limits for the critical line 471 Hill – 435 Sycamore, were determined by sensitivity analysis. These results compare accurately with those obtained from repetitive application of the TEF method and the power flow studies(Table 6.6).

From table 6.6 it can be seen that, in the worst case, a 11 MW decrease in power flow at 471 Hill – 435 sycamore (positive line power flow direction is from Hill to Sycamore) will make the system critically stable assuming that generation shifts occur at four critical generators.

To show the validity of rule 3 in section 4.3 for selecting a critical line, three non-critical lines are chosen. The sensitivity analysis for these lines are shown in Table 6.7. These results clearly show the effect of the distribution factors on the sensitivity analysis for these lines.

Table 6.6: Line Power Flow Limits at Line 471 Hill - 435 Sycarmore

| Contingency | | | Range of Line Power Flow Limits | | | |
|---|---|---|---|---|---|---|
| Fault Loc. | Clearing time(sec) | Critical Generator | Sensitivity Analysis | Full TEF Results | Error (%) | |
| | | | | | Low.Lim. | Upp.Lim. |
| Ft.Cal. | 0.342 | $G_{10}$ $G_{12}$ $G_{16}$ $G_{17}$ | $-16.89 \leq \Delta I_l \leq -11.40$ | $-16.22 \leq \Delta I_l \leq -11.09$ | 4.1 | 2.8 |
| C.Blff. | 0.187 | $G_{10}$ $G_{12}$ | $-14.35 \leq \Delta I_l \leq -11.86$ | $-14.27 \leq \Delta I_l \leq -11.75$ | 0.6 | 0.9 |
| Cooper | 0.191 | $G_2$ | $\Delta I_l = -17.02$ | $\Delta I_l = -16.62$ | 2.3 | |

Table 6.7: Line Power Flow Limits at Non-critical Line

| Monitored Line | Contingency | Range of Line Power Flow Limits | | | |
|---|---|---|---|---|---|
| | | Sensitivity Analysis | Full TEF Results | Error (%) | |
| | | | | Low.Lim. | Upp.Lim. |
| 193 Lakefd | Ft.Cal. | $-6.1 \leq \Delta I_l \leq -4.83$ | $-5.67 \leq \Delta I_l \leq -4.59$ | 7.6 | 5.2 |
| – | C.Blff. | $-5.27 \leq \Delta I_l \leq -4.35$ | $-5.06 \leq \Delta I_l \leq -4.19$ | 4.2 | 3.8 |
| 372 Raun | Cooper | $\Delta I_l = -5.89$ | $\Delta I_l = -5.46$ | 7.9 | |
| 7 Lincoln | Ft.Cal. | $-6.89 \leq \Delta I_l \leq 0.39$ | $-6.64 \leq \Delta I_l \leq 0.41$ | 3.8 | -4.9 |
| – | C.Blff. | $-2.38 \leq \Delta I_l \leq -2.10$ | $-2.37 \leq \Delta I_l \leq 2.07$ | 0.4 | 1.4 |
| 779 S.3454 | Cooper | $\Delta I_l = 3.97$ | $\Delta I_l = 3.85$ | 3.1 | |
| 6 Cooper | Ft.Cal. | $6.59 \leq \Delta I_l \leq 12.64$ | $6.6 \leq \Delta I_l \leq 12.33$ | -0.1 | 2.5 |
| – | C.Blff | $6.61 \leq \Delta I_l \leq 7.29$ | $6.74 \leq \Delta I_l \leq 7.55$ | -1.9 | 3.4 |
| 393 S.Joe | Cooper | $\Delta I_l = 15.30$ | $\Delta I_l = 15.07$ | 1.5 | |

(Critical Generators for each Contingency)

Ft.Cal. Fault : $G_{10}$, $G_{12}$, $G_{16}$, $G_{17}$

C.Blff. Fault : $G_{10}$, $G_{12}$

Cooper Fault : $G_2$

# 7 CONCLUSION

This dissertation used the transient energy function technique to relate the energy margin with relevant system parameters. The sensitivity of the energy margin with respect to generation shift has been used to determine plant generation loading limits and interface power flow limits. By relating the energy margin with important system parameters, the TEF method gives qualitative and quantitative description of the change in the power system transient stability behavior.

The proposed procedure was validated by comparing the sensitivity analysis results and full TEF program runs. The following conclusions can be drawn from the data presented in this dissertation.

1. An analytic technique has been developed to relate the transient energy margin with generation shifts and interface power flow limits. Sensitivity analysis successfully predicts the generator loading limits and line power flow limits with reasonable accuracy when the system is stability limited.

2. The sensitivities of the energy margin with respect to generation shifts are accurate when generation shifts occur between critical group of generators and the reference machine (The reference machine belongs to non-critical group of generators.).

3. Transmission interface power flow limits are not unique for the interconnected system. With a reasonable assumption, the range of transmission interface power flow limits can be obtained by sensitivity analysis and linear programming.

4. Sensitivity analysis on the TEF method on the base power flow case allows determination of secure operating conditions for various conceivable contingencies. It provides needed generation shifts for secure operation of power system when the system is stability limited.

## 7.1 Suggestions for Future Work

The experiences gained during the research project suggest the following subjects of investigation.

1. To investigate the case where the Mode of Disturbance is changing due to generation shifts.

2. To investigate the second order sensitivity.

3. To extend analytic sensitivity technique to higher order power system models.

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# 10 APPENDIX A

To obtain approximated value of $\frac{\partial|E_i|}{\partial P_{mk}}$, two power flow solutions are needed for each generation change. To avoid this process a simplified method is developed.

Because the internal voltage of the generator whose generation is changing varies much more than the other generators, $\frac{\partial|E_i|}{\partial P_{mk}}$ $(i \neq k)$ are assumed to be zero. Only $\frac{\partial|E_k|}{\partial P_{mk}}$ is approximated using the Kirchhoff's law.

<u>Algorithm to approximate $\frac{\partial|E_k|}{\partial P_{mk}}$ due to generation change</u>

When there is generation change, it is assumed that only real power is changing. But in actual power flow, generator which participate in generation change also changes its reactive power. To take this into account, reactive power change $\Delta Q_k$ is considered. The ratio of $\frac{\Delta P_{mk}}{\Delta Q_k}$ is picked by observing the actual power flow.

Figure 10.1 shows the generator terminal branch which participate in generation change for the base power flow case. If there is a complex power change $(\Delta P_{mk} + \Delta Q_k)$ at the machine terminal with the terminal voltage $V_k$ held con-

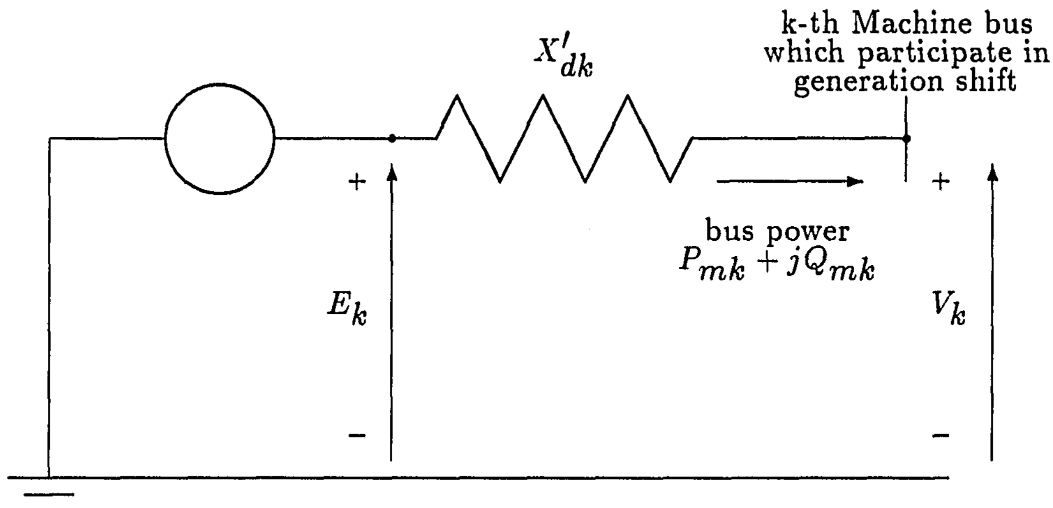


Figure 10.1: Equivalent Circuit of Classical Machine Model during Transient

stant [1], then $E_k$ will change to $E'_K$. By applying Kirchhoff's law at this branch, we have,

$$E'_k = -\frac{P_{mk} + \Delta P_{mk} - j(Q_k + \Delta Q_k)}{V_k^*}(jX'_d) + V_k \tag{10.1}$$

Then $\frac{\partial |E_k|}{\partial P_{mk}}$ can be approximated as

$$\frac{\partial |E_k|}{\partial P_{mk}} \approx \frac{|E'_k| - |E_k|}{\Delta P_{mk}} \tag{10.2}$$

For the 17 Generator Iowa System several values of $\Delta Q$ were tried and $\Delta Q = \frac{Q_k}{P_k}\Delta P_{mk}$ showed good results.

[1]In power flow studies active power and terminal voltage of generator node are held constant

# 11 APPENDIX B

Problem in section 4.1 and 4.2 can be transformed into following form without loss of generality.

Problem

Minimize and maximize

$$|\sum_{i=1}^{n} a_i x_i| \tag{11.1}$$

Subject to

$$\sum_{i=1}^{n} x_i = -c$$
$$-c \leq x_i \leq 0 \quad for \quad all \quad i$$

where

$0 < c$

$a_i$ are of the same sign for all $i = 1, 2, ..., n$

Solution

$$min \quad |\sum_{i=1}^{n} a_i x_i| = a_* c$$

$$max \quad |\sum_{i=1}^{n} a_i x_i| = a^* c$$

where

$a_* = \; min \quad \{|a_i|\}$

$a_* = \; max \quad \{|a_i|\}$

Proof

Note that feasible value of $\underline{x} = (x_1, x_2, ..., x_n)$ should lie on the set $\Omega$ .

$$\Omega = \{\tau_1(-ce_1) + \tau_2(-ce_2) + ... + \tau_n(-ce_n) | 0 \le \tau_i \le 1 \;\; and \;\; \sum_{i=1}^{n} \tau_i = 1\}$$

where $e_i$ is the unit vector whose i-th component is 1 and zero for other components.

Let $V = \{(-ce_1), \; (-ce_2), \; ... \; (-ce_n)\}$ is a set of extreme point of $\Omega$. Let us assume $\underline{x} \notin V$ and achieve its extremum of equation (11.1). Since $\sum_{i=1}^{n} a_i x_i$ is differentiable in the domain $\Omega$ and achieve its extremum, it is necessary to satisfy $\frac{\partial}{\partial \underline{x}}(\sum_{i=1}^{n} a_i x_i) = 0$ . But $\frac{\partial}{\partial \underline{x}}(\sum_{i=1}^{n} a_i x_i) = (a_1, \; a_2, \; ... \; , a_n) \neq 0$ . It contradicts the assumption. Therefore $\underline{x}$ which achieves extremum of $\frac{\partial}{\partial \underline{x}}(\sum_{i=1}^{n} a_i x_i)$ should be contained in $V$ .

But inner product of $(a_1, \ a_2, \ \dots \ , a_n)$ and one of elements of set V gives $-ca_i$. Hence solutions of the problem are as follows.

$$min \ \ |\sum_{i=1}^{n} a_i x_i| = a_* c$$

$$max \ \ |\sum_{i=1}^{n} a_i x_i| = a^* c$$